Physical vs Digital Currency
A Difference that Makes a Difference*

Nicola Amendola† Luis Araujo‡ Leo Ferraris§

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Abstract

This paper compares digital and physical currency, focusing on a single intrinsic difference: digital, unlike physical currency, allows the monetary authority to trace the monetary flows in and out of the accounts. We show that this technological advance in record-keeping can be used by the monetary authority to improve upon physical currency and achieve efficiency in a wide range of circumstances.

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†Department of Economics and Finance, University of Rome, Tor Vergata

‡Department of Economics - MSU, and Sao Paulo School of Economics-FGV

§Department of Economics, Management and Statistics, University of Milan-Bicocca

1 Introduction

In academic and policy circles, there is an active debate about the implications of introducing sovereign digital currency as a payment instrument alongside or instead of physical currency. The literature has identified a number of benefits and costs of digital currency. Benefits include lower transaction costs, increased competition in the banking sector, and lower tax evasion. Costs include privacy breach, infringement of personal freedom, and risk of disintermediation. Neither the benefits nor the costs are strictly connected with the monetary nature of the instrument, which is often conceived simply as an intangible version of cash.

In this paper, we identify a single intrinsic difference between digital and physical currencies and show how it matters for optimal monetary policy. To make it transparent that this is the only driving force, we contrast two pure currency economies that are identical except for the type of currency that is used, either physical or digital, comparing their functioning and identifying the optimal monetary policy in each case. The only difference between the two instruments is that with digital currency the monetary authority can keep track of flows of money balances in and out of the accounts, while with physical currency this is not possible.[1] The extra information collected through the digital technology can always be ignored, hence, any allocation attainable with physical currency can be reached with digital currency, but there are robust circumstances in which digital currency improves strictly upon cash.

We consider a monetary search economy à la Lagos and Wright (2005), in which the velocity of circulation of currency is endogenous, since the traders choose their search intensity, as in Rocheteau and Wright (2005). In the basic framework, we adopt the matching technology of Lagos and Rocheteau (2005) and Kalai's bargaining protocol. We characterize the equilibrium and identify the optimal intervention by the monetary authority with physical and digital currencies, respectively. With physical currency, the optimal intervention is the Friedman rule, which, however, does not always achieve efficient search intensity. Moreover, any deviation from the Friedman rule always reduces the search intensity and the velocity of money. With digital currency, instead, when the equilibrium search intensity is below its efficient level,

[1] This difference in the record-keeping possibilities with different types of payment instruments is in the spirit of Kocherlakota (1998). A real world counterpart would be the digital euro that is being designed by the ECB with traceability of digital currency flows; See the website of the ECB on the digital euro.

efficiency can be restored, paying interest to stimulate the velocity of circulation of currency, with a policy that deviates from the Friedman rule. When the equilibrium search intensity is excessive relative to efficiency, the allocation can still be improved paying interest on idle balances to reduce the velocity of circulation of currency, thus discouraging participation. Except in the knife-edge case that corresponds to the Hosios condition of labor economics, the additional information available with digital currency makes a difference for optimal policy.

We show that the result holds in a robust set of circumstances, beyond the specific trading arrangement adopted in the basic framework. Following Gu and Wright (2016), we generalize the trading protocol to a mechanism that subsumes, among other schemes, Kalai and Nash's bargaining protocols. We also show that the result survives when the taxation of balances is not feasible, as in Hu, Kennan, and Wallace (2009). Finally, the result continues to hold if the agents can try to manipulate the system at a cost, opening "shadow accounts" to obtain interest on balances that are not truly active.

There is a large literature on monetary models with trading externalities and extensive margins, going back to Li (1994, 1995), including, among others, the work of Lagos and Rocheteau (2005), Liu, Wang and Wright (2011), and Hu and Zhang (2019). The book by Nosal and Rocheteau (2011) contains a thorough discussion of the issues at stake. A growing literature examines the impact of digital currencies. For example, recent papers that adopt the Lagos and Wright (2005) framework include Williamson (2022), Chiu et al. (2023), and Keister and Sanches (2023). In these papers, interest can be paid with digital currency unlike with physical currency but *uniformly* on all balances.[2] Restricting the scheme to the payment of interest uniformly on all balances, both active and idle, digital currency cannot enlarge the set of implementable allocations relative to physical currency, as the optimal allocation coincides with the one achieved under the Friedman rule in the corresponding physical currency economy. However, uniform interest is suboptimal, as it does not take into account the additional information about monetary flows available through digital currency, which helps alter the velocity of currency as required by efficiency.

The welfare enhancing role of paying interest on currency has been examined in the monetary literature by Andolfatto (2010), Wallace (2014), and Bajaj et al.

[2]A scheme with uniform interest payment on privately issued digital currency is also examined by Chiu and Wong (2022).

(2017), among others. It is well known that interest-bearing money is not essential if lump-sum taxes are available. In particular, the Friedman rule can be achieved with a zero nominal interest rate if the monetary authority can implement a deflation financed by a lump-sum tax that contracts the money supply at the appropriate rate. A key novelty of our approach consists in showing that this result applies to physical currencies but does not extend to digital currencies when extensive margin considerations matter. This is because in a digital currency economy the monetary authority can exploit the traceability of digital currency to directly affect the extensive margin by rewarding active or idle balances with interest as needed. A lump-sum tax instrument, being unconditional, cannot alter participation and affect the extensive margin in such a manner. The implication is that, with digital currency, interest-bearing money is essential even if a lump-sum tax instrument is available.

The paper proceeds as follows. Section 2 introduces the model. Section 3 presents the results, and Section 4 compares them with the literature. Section 5 presents three extensions, and Section 6 concludes. The appendix contains the omitted proofs.

2 Model

The model is a version of Lagos and Wright (2005) with an endogenous participation decision, as in Rocheteau and Wright (2005) and Lagos and Rocheteau (2005). In this section, we describe the fundamentals, how trade occurs, and derive the efficient benchmark.

2.1 Fundamentals

Time is discrete. Each period is divided into two sub-periods, called day and night. There is a continuum of infinitely lived agents who discount the future at a rate $\beta \in (0, 1)$. At night, all agents meet in a centralized Walrasian market where a commodity is traded that serves as the unit of account and can be produced and consumed by all agents, with linear payoffs. During the day, agents meet in a decentralized market to trade another commodity, whose non-negative quantity is denoted by q. A measure π_s of these agents are sellers of this commodity that, to be produced, requires a cost represented by a twice differentiable, strictly increasing, and convex function $c(q)$. In turn, a measure π_b of the agents are potential consumers of this product. Consumption of the commodity gives utility represented by a twice differentiable strictly increasing and strictly concave function, $u(q)$. We assume $u(0) = c(0) = u'(\infty) = c'(0) = 0$, and $u'(0) = \infty$. Define the net gain during the day as $U(q) \equiv u(q) - c(q)$. Potential consumers choose the search intensity $\alpha \in [0, 1]$, incurring a cost represented by a twice differentiable, strictly increasing, and strictly convex function $k(\alpha)$. We assume $k(0) = k'(0) = 0$. We denote by $\overline{\alpha}$ the average search intensity of buyers.

2.2 Trade

During the day, in the decentralized market, buyers and sellers meet bilaterally according to the matching function $\zeta(\overline{\alpha}\pi_b, \pi_s)$ that is homogeneous of degree one, twice continuously differentiable, strictly increasing and strictly concave in each argument. Suppose also that $\zeta(0, \pi_s) = \zeta(\bar{\alpha}\pi_b, 0) = 0$ and $\zeta(\overline{\alpha}\pi_b, \pi_s) \leq \min\{\pi_b, \pi_s\}$ for any $\overline{\alpha} \geq 0$. Define $\zeta(\overline{\alpha}\pi_b, \pi_s)/\overline{\alpha}\pi_b \equiv \mu(\pi_s/\overline{\alpha}\pi_b)$. We assume that $\mu(\cdot)$ takes values in the unit interval for any value of $\pi_s/\overline{\alpha}\pi_b$ and approaches unity when $\pi_s/\overline{\alpha}\pi_b$ diverges to infinity.[3]. The probability of a buyer individual meeting is given by $\alpha\mu(\pi_s/\overline{\alpha}\pi_b)$. In what follows, we normalize the population sizes to one, that is, $\pi_b = \pi_s = 1$. The elasticity of the matching function with respect to the average participation rate $\overline{\alpha}$ is given by the function

$$\epsilon(1/\overline{\alpha}) \equiv 1 - \frac{\mu'(1/\overline{\alpha})}{\overline{\alpha}\mu(1/\overline{\alpha})},$$

that takes values in the unit interval. To ensure that the efficient benchmark is well defined, we assume that $\epsilon(\cdot)$ is not decreasing in its argument, hence not increasing in $\overline{\alpha}$. The terms of trade within the meetings are determined with the Kalai protocol, where $\theta \in (0, 1]$ denotes the bargaining power of the buyer.[4] Traders cannot commit to future actions and are anonymous during the day. Trades are not observable by outsiders.

2.3 Efficiency

Due to the linearity of the payoffs, utility is transferable at night and has no impact on welfare. Hence, we focus on day-time decisions. The efficient allocation is a participation rate α and a day quantity q that maximize $\alpha\mu(1/\alpha)U(q) - k(\alpha)$ subject

[3] This is the same matching function used by Lagos and Rocheteau (2005).

[4] See Kalai (1977). Lagos and Wright (2005) used the Nash protocol. In section 5.3, we show that our results hold in a larger class of trading protocols, including Nash bargaining.

to $\alpha \leq 1$. To avoid having to deal with the corner solution for the participation rate in efficient allocation, we make the following assumption.[5]

Assumption 1 $k'(1) > \mu(1)\epsilon(1)U(q^*)$.

The optimality condition for α is

$$\mu(1/\alpha)\epsilon(1/\alpha)U(q) = k'(\alpha), \tag{1}$$

that equates the expected gains from trade to the marginal cost of participation. The optimality condition for q is

$$\alpha\mu(1/\alpha)U'(q) = 0, \tag{2}$$

that equates the expected marginal benefit and the cost of production. Efficient allocations, denoted with a star, satisfy equations (1) and (2). For any positive participation rate, there is a unique q^* that satisfies $U'(q) = 0$, since $U'(0) = \infty$, $U'(\infty) < 0$ and $U''(q) < 0$, by the properties of the fundamentals. The difference in LHS and RHS of equation (1) with $q = q^*$ has the following characteristics. First, it is strictly positive for $\alpha = 0$, by the properties of the matching function. Second, it is negative for $\alpha = 1$, by Assumption 1. Finally, it strictly decreases in α, by the properties of the matching and cost functions. Hence, a unique positive $\alpha^* < 1$ satisfies (1). We conclude that there exists a unique efficient allocation (α^*, q^*). Henceforth, we shall denote $\epsilon^* \equiv \epsilon(1/\alpha^*)$.

3 Physical vs Digital Currency

The lack of monitoring and limited commitment prevent trade during the day in the absence of a trading instrument. We consider two alternative economies, one in which the trading instrument is physical currency and another with digital currency. These economies are identical except for the use of the trading instrument. In particular, in both economies, currency is introduced through accounts open by the monetary authority for each agent at the beginning of the first period. We distinguish trading instruments by the record-keeping technology embedded in the accounts. In the physical currency economy, the monetary authority only keeps track of the current balance

[5]Later in the paper, we examine the opposite assumption. The main result extends to this case.

in the account, whereas in the digital currency economy the monetary authority keeps track of both the current balance and all past balances.[6] In both economies, we restrict attention to stationary symmetric equilibria with time-invariant real balances. Throughout, we denote by ϕ the price of the currency in the night market.

3.1 Physical Currency

We begin with the physical currency economy. We characterize the equilibrium and then determine the optimal policy.

3.1.1 Equilibrium

Consider meetings between buyers and sellers in the day market. Under Kalai bargaining, buyers holding an amount m of currency choose a payment $d \leq m$ in exchange for a quantity q to maximize $u(q) - \phi d$, subject to the constraint

$$(1-\theta)[u(q) - \phi d] = \theta[\phi d - c(q)]. \tag{3}$$

Note that the real balances of the sellers do not affect the terms of trade. Moreover, since sellers have no use for real balances in the day market, in equilibrium it is never the case that they strictly prefer to bring currency. We assume that buyers spend all their balances during the day. In the following, we will show that this is, in fact, the optimal choice of buyers. This implies $d = m$, and we can rewrite (3) as

$$\phi m = (1-\theta)u(q) + \theta c(q) \equiv g(q), \tag{4}$$

[6] The assumption that the monetary authority only tracks the current balance in the account in the physical currency economy implies that policy can depend solely on the current balance and not on past balances; that is, information released by the record-keeping technology in previous periods cannot be used. An equivalent alternative would be to assume that agents do not have accounts at the central bank in the physical currency economy. In that case, given agents' anonymity, even if the monetary authority observes an agent's current balance in each period, it cannot identify the same agent in multiple periods and, therefore, cannot implement policies that condition on information about past balances. We adopted the former specification only to facilitate a direct comparison between the physical and digital currency economies. Moreover, if the monetary authority can levy lump-sum taxes, the actual observation of current balances in the physical currency economy is not necessary for our results. As shown in Section 4.2, this assumption is introduced to facilitate comparison with Andolfatto (2010), Wallace (2014), and Bajaj et al. (2017), which study policies in physical currency economies when lump-sum taxes are not feasible.

which determines the quantity produced by sellers as a function of buyers' real balances.

Consider now the participation decision of buyers with real balances ϕm at the beginning of the day. Choosing α, buyers incur the cost $k(\alpha)$ and meet sellers with probability $\alpha\mu(1/\overline{\alpha})$, in which case they spend all their balances. If buyers do not meet sellers, they keep their balances. Buyers choose $\alpha \leq 1$ to maximize $\alpha\mu(1/\overline{\alpha})u(q)+[1-\alpha\mu(1/\overline{\alpha})]\phi m - k(\alpha)$. Using (4), and ignoring for the moment the corner solution for the participation rate, we obtain the following optimality condition for participation

$$\mu(1/\overline{\alpha})\theta U(q) = k'(\alpha), \tag{5}$$

which equates the expected buyers' surplus to the marginal cost of participation. We now move to the night market and consider the buyers' choice of the amount of balances to bring into the day market. Buyers choose m to maximize $-\phi m + \beta\{-k(\alpha) + \alpha\mu(1/\overline{\alpha})u(q) + [1 - \alpha\mu(1/\overline{\alpha})]\phi_{+1}m\}$. Using equation (4), we obtain the inter-temporal condition for the optimum

$$\phi = \beta\phi_{+1}\left[\alpha\mu(1/\overline{\alpha})\frac{u'(q)}{g'(q)} + 1 - \alpha\mu(1/\overline{\alpha})\right]. \tag{6}$$

In words, an extra unit of currency acquired presently can be spent next period on day consumption if the agents turn out to be buyers and are matched to sellers or kept idle until the night market otherwise.

In an economy with physical currency, the monetary authority can inject or tax currency in a lump-sum fashion. If we let M denote the quantity of currency and τ denote the growth rate of currency stock, we have $M_{+1} = (1+\tau)M$. The stationarity of real balances implies $\phi/\phi_{+1} = M_{+1}/M = 1+\tau$, and we can rewrite (6) as follows

$$\frac{u'(q)}{g'(q)} = \frac{\tau + 1 - \beta + \alpha\mu(1/\overline{\alpha})\beta}{\alpha\mu(1/\overline{\alpha})\beta}. \tag{7}$$

The existence of a monetary equilibrium requires $\tau \geq \beta - 1$ to prevent an infinite demand for money. This condition also ensures that it is never strictly optimal for the buyers to bring balances they do not plan to use, hence vindicating our initial assumption that the buyers spend all their balances. A stationary symmetric physical currency equilibrium is a pair (α, q), such that $\alpha = \overline{\alpha}$ and (5) and (7) are satisfied for $\tau \geq \beta - 1$. The existence of equilibrium for any feasible policy is established as in Lagos and Rocheteau (2005), giving $(\alpha(\tau), q(\tau))$.

3.1.2 Optimal Policy

The right-hand side of (7) increases in τ. Since the function $u'(q)/g'(q)$ decreases in q, due to the strict concavity of the utility function and the convexity of the cost function, $q'(\tau) < 0$. Since $g'(q) = u'(q) - \theta U'(q)$, by the equilibrium condition (7), the efficient quantity q^* is only achieved if we set $\tau = \beta - 1$. This is the policy known as the Friedman rule. The participation condition (5) is affected by policy only indirectly and the participation rate decreases with policy, $\alpha'(\tau) < 0$. If $\theta = \epsilon^*$, conditions (5) and (1) coincide at the Friedman rule and therefore equilibrium participation is efficient, i.e., $\alpha = \alpha^*$. In this non-generic case, which corresponds to what is known in the labor literature as the Hosios condition, buyers choose to participate in the exact right amount so that the first-best is achieved with $\tau = \beta - 1$. When $\theta \neq \epsilon^*$, at the Friedman rule that achieves $q = q^*$, conditions (5) and (1) do not coincide, and by the properties of the functions $k'(\alpha)$ and $\mu(1/\alpha)$ it immediately follows that equilibrium participation is inefficient, i.e., $\alpha \neq \alpha^*$. In particular, since $k'(\alpha)/\mu(1/\alpha)$ is increasing monotonically in α, if $\theta < \epsilon^*$, participation is inefficiently small, i.e., $\alpha < \alpha^*$, while if $\theta > \epsilon^*$, it is inefficiently large, i.e., $\alpha > \alpha^*$. The next Lemma summarizes the result.

Lemma 1 *Efficiency at the intensive margin is only achieved at the Friedman rule. If $\theta = \epsilon^*$, the Friedman rule achieves efficiency in the intensive and extensive margins; if $\theta \neq \epsilon^*$, the Friedman rule achieves efficiency only in the intensive margin: for $\theta < \epsilon^*$, participation is inefficiently low, for $\theta > \epsilon^*$, inefficiently high.*

The Friedman rule drives the intensive margin to efficiency rewarding balances at the rate of time preference to compensate for the elapse of time. However, it is unable to drive the participation decision towards full efficiency, as it lacks the tools to stimulate or discourage buyers' participation.[7] When $\theta < \epsilon^*$, there is a thick market externality at work, according to which an increase in the search intensity of buyers increases the matching probability of sellers. When $\theta > \epsilon^*$, there is a congestion externality at work, according to which an increase in the search intensity of buyers decreases their

[7] To emphasize the role of trading externalities in preventing the Friedman rule from achieving full efficiency, in the Appendix we consider an economy where the decentralized market is replaced by a Walrasian market where there is no externality. The Friedman rule achieves the first-best.

matching probability. In order to internalize these externalities, the buyers would need to receive a share of the surplus commensurate with their contribution to the matching process, which requires $\theta = \epsilon^*$.

3.2 Digital Currency

With digital currency, record-keeping technology allows the monetary authority to track the flows of balances in and out of the accounts. In what follows, we use this information to separate balances in the end-of-the-day market into two groups, labeled passive and active. A balance in an account at the end of the day market is passive if it was already in the account at the beginning of the day market; while a balance in an account at the end of the day market is active if it was transferred into the account during the day market. We consider interventions where the monetary authority treats these balances differently, offering two distinct nominal interest rates, i_p for passive balances and i_a for active balances. We will also use the net interest $i \equiv i_a - i_p$. We proceed by first determining the equilibrium and then characterizing the optimal policy.

3.2.1 Equilibrium

Consider a meeting between sellers and buyers with m units of digital currency in the day market. If buyers transfer d balances to sellers and receive q units of goods, the buyer's surplus is $S_b = u(q) - \phi(1 + i_p)d$, while the seller's surplus is $S_s = -c(q) + \phi(1 + i_a)d$. Observe that the buyer's surplus includes the interest lost on the currency that was transferred to sellers, and correspondingly, the seller's surplus includes the interest gained in the process. The total surplus is $S = U(q) + \phi id$, that is, the gain from trade and the net interest payment to the transferred balances.

With Kalai bargaining, buyers holding an amount m of currency choose $d \leq m$ in exchange for a quantity q to maximize $u(q) - \phi(1 + i_p)d$ subject to the constraint $(1 - \theta)S_b = \theta S_s$. As with physical currency, the real balances of the sellers do not affect the terms of trade. However, sellers may want to bring real balances to the day market, depending on the nominal interest rate paid on idle balances. We will examine this incentive below, showing that it is never part of the optimal policy to give sellers the incentive to bring balances into the day market. We will also show that, as in the case of physical currency, buyers spend all their balances during the day. This allows us to rewrite the constraint $(1-\theta)S_b = \theta S_s$ as

$$\phi m \equiv \frac{g(q)}{1+i_p+\theta i}. \tag{8}$$

Consider now the participation decision of buyers with balances m at the beginning of the day market. Choosing α, buyers incur a cost $k(\alpha)$, meet sellers with probability $\alpha\mu(1/\overline{\alpha})$, and spend all their balances. If buyers do not meet sellers, they keep their balances until the night market and receive an interest payment i_p. Formally, buyers choose $\alpha \leq 1$ to maximize $\alpha\mu(1/\overline{\alpha})u(q) + [1-\alpha\mu(1/\overline{\alpha})](1+i_p)\phi m - k(\alpha)$. Define $\delta(q) \equiv u(q)/U(q)$ and $F(q,i,i_p) \equiv [1+i_p+\delta(q)i]/(1+i_p+\theta i)$. Using (8) and ignoring for the moment the corner solution for the participation rate, we obtain the optimality condition for participation

$$\mu(1/\overline{\alpha})\theta F(q,i,i_p)U(q) = k'(\alpha), \tag{9}$$

which equates the expected buyers' surplus to the marginal cost of participation. We now move to the night market. Buyers choose m to maximize $-\phi m+\beta[\alpha\mu(1/\overline{\alpha})u(q)+(1-\alpha\mu(1/\overline{\alpha}))(1+i_p)\phi_{+1}m-k(\alpha)]$. Using (8), we obtain the inter-temporal condition for the optimum

$$\phi = \beta\phi_{+1}\left\{\alpha\mu(1/\overline{\alpha})\frac{u'(q)}{g'(q)}(1+i_p+\theta i) + (1-\alpha\mu(1/\overline{\alpha}))(1+i_p)\right\}. \tag{10}$$

In words, an extra unit of currency acquired presently can be spent next period on day consumption if the agents turn out to be buyers and are matched to sellers or kept idle until the night market otherwise. In either case, the agents receive interest payments for actively using the balances or keeping them idle.

The monetary authority can inject or tax currency in a lump-sum manner. It can also inject or tax currency using the nominal interest rates i_p and i. If we let τ denote the rate at which money is injected or taxed in the economy by lump sum, we have $M_{+1} = [1+\tau+i_p+\alpha\mu(1/\overline{\alpha})i]M$. The stationarity of real balances implies $\phi/\phi_{+1} = M_{+1}/M$. Therefore, we can rewrite (10) as

$$\frac{u'(q)}{g'(q)} = \frac{\tau+[1-\beta+\alpha\mu(1/\overline{\alpha})\beta](1+i_p)+\alpha\mu(1/\overline{\alpha})i}{\alpha\mu(1/\overline{\alpha})\beta\,(1+i_p+\theta i)} \equiv G(\alpha,\tau,i,i_p). \tag{11}$$

The existence of a monetary equilibrium requires $\phi \geq \beta\phi_{+1}(1+i_p)$, otherwise an agent would have the incentive to demand an infinite amount of currency. Thus, we obtain the following condition

$$\tau \geq -[(1-\beta)(1+i_p) + \alpha\mu(1/\overline{\alpha})i]. \tag{12}$$

A stationary symmetric digital currency equilibrium is a pair (α, q) such that $\alpha = \overline{\alpha}$ and (9) and (11) are satisfied for policy (τ, i, i_p) that satisfies (12). In the appendix, we provide sufficient conditions for the existence of an equilibrium.

An increase in τ has no direct impact on the participation of buyers; but it reduces the surplus, with a negative impact on the quantity produced in trade meetings. In contrast, changes in the nominal interest rates on active and idle balances impact both the extensive and the intensive margins. Let us compare these effects with the physical currency economy in which there are no interest payments.

We start with the intensive margin. Fixing the extensive margin, an increase in interest rates leads to money creation, which negatively impacts the real rate of return on balances. This reduces the demand for real balances and the quantity in trade meetings, as captured in the numerator of $G(\alpha, \tau, i, i_p)$. However, an increase in interest rates increases the quantity traded for any given real balances. This is so because an increase in interest rates increases the trade surplus, which leads to an increase in the quantity produced in the meetings. This positive effect is captured in the denominator of $G(\alpha, \tau, i, i_p)$. The overall effect is ambiguous.

Consider now the extensive margin. Fixing the intensive margin, if the net interest payment is nil $F(q, 0, i_p) = 1$. Thus, rewarding idle balances alone cannot stimulate participation. Instead, if the interest payment on the passive balances is nil but $i > 0$, we have $F(q, i, 0) > 1$, since $\delta(q) > 1 \geq \theta$. Thus, an interest payment on active balances has a positive impact on the extensive margin. It does so by increasing the participation of buyers. Intuitively, an increase in i encourages participation due to its direct positive effect on trade surplus.

3.2.2 Optimal Policy

In the digital currency economy, in addition to impacting the overall return on currency through changes in τ, the monetary authority can use i and i_p to target how the return on currency will be distributed between agents holding idle balances and agents holding active balances. In what follows, we will show that the availability of these additional instruments allows the monetary authority to implement the first-best in many circumstances. A policy is a triple (τ, i, i_p). Consider first the knife-edge case with $\theta = \epsilon^*$. Since $F(q^*, 0, 0) = 1$, first-best is achieved with $\tau = \beta - 1$ and $i = i_p = 0$ as in the physical currency economy.

Proposition 1 *If $\theta = \epsilon^*$, the Friedman rule achieves the first-best.*

When the Hosios condition is satisfied, efficiency is obtained without interest payment through the Friedman rule. Next, we distinguish two cases on either side of the Hosios condition, the case in which the thick market externality prevails, with $\theta < \epsilon^*$, and the case in which the congestion externality prevails, with $\theta > \epsilon^*$.

3.2.2.1 Thick Market Externality

Let us begin with the case in which the thick market externality prevails, with $\theta < \epsilon^*$. Since $g'(q) = u'(q) - \theta U'(q)$, to induce efficient production q^* within trade meetings, given the interest payments, the monetary authority sets τ so that $G(\alpha^*, \tau, i, i_p) = 1$, that is,

$$\tau = -[(1 - \beta)(1 + i_p) + \alpha^* \mu(1/\alpha^*)(1 - \beta\theta)\, i]. \tag{13}$$

Using (13), the condition (12) with $\alpha = \alpha^*$, which guarantees the existence of monetary equilibrium, boils down to $i \geq 0$. Regarding efficient participation, given τ, the monetary authority sets the interest payments to ensure that the equilibrium participation condition (9) replicates the efficient participation condition (1), both evaluated in efficient allocation (α^*, q^*). Given τ, the monetary authority sets the interest payments (i, i_p) to achieve the following condition

$$\theta F(q^*, i, i_p) = \epsilon^*. \tag{14}$$

The idea is that the nominal interest payments should compensate whenever possible for the insufficient bargaining power of buyers, increasing the trade surplus as needed to achieve the first best.

A natural candidate for the implementation of the first-best through equation (14) has $i_p = 0$ and the optimal net interest payment

$$i^* = \frac{\epsilon^* - \theta}{\theta[\delta(q^*) - \epsilon^*]}, \tag{15}$$

which is positive since $\theta < \epsilon^*$ and $\delta(q^*) > 1 > \epsilon^*$. Using (13), we obtain

$$\tau^* = \beta - 1 - \frac{\alpha^* \mu(1/\alpha^*)(1-\beta\theta)(\epsilon^* - \theta)}{\theta[\delta(q^*) - \epsilon^*]}. \tag{16}$$

Since $i^* > 0$, condition $i \geq 0$ is satisfied. Moreover, $i^* > 0 = i_p^*$ implies $\phi > \beta\phi_{+1}$, so the economy is away from the Friedman rule. There is money creation under the optimal policy if $\tau^* + \alpha^* \mu(1/\alpha^*) i^* > 0$, which can be rewritten as

$$\beta > \frac{\delta(q^*) - \epsilon^*}{\delta(q^*) - \epsilon^* + \alpha^* \mu(1/\alpha^*)(\epsilon^* - \theta)},$$

where the lower bound is smaller than 1 since $\delta(q^*) > 1 > \epsilon^*$ and $\theta < \epsilon^*$. The next proposition summarizes the result.

Proposition 2 *If $\theta < \epsilon^*$, there exist policy schemes that deviate from the Friedman rule and implement the first-best rewarding active balances with positive interest and passive balances with zero interest.*

This is the main result of the paper. When there are distortions that would imply inefficiencies in both intensive and extensive margins, the optimal monetary policy in the physical-currency economy, which is the Friedman rule, achieves efficiency along the intensive, but not the extensive margin. In the corresponding digital currency economy, the optimal policy also achieves efficiency along the extensive margin, stimulating the participation of the buyers in the process of trade through the payment of interest on active but not idle balances when the thick market externality prevails. As can be seen in (10), the optimal policy is different from the Friedman rule. In fact, if the authority chooses the Friedman rule, efficiency along the intensive margin requires $i = 0$, otherwise there would be an infinite demand for money. However, $i = 0$ does not incentivize the participation of buyers, which is inconsistent with efficiency at the extensive margin if there is a thick market externality. Unlike in the physical currency economy, the optimal policy is feasible in the digital currency economy, since the technology allows the authority to observe the flows of balances in and out of the accounts, opening up the possibility to distinguish active and idle balances.

Clearly, the optimal interest rate on active balances (15) is a decreasing function of θ. When buyers have more bargaining power in negotiations with sellers, they acquire a larger share of the gains from trade, which increases their willingness to participate in trade. Therefore, a smaller interest payment is needed to achieve efficiency.

Another interesting result of comparative statics concerns productivity. Suppose that the cost function is scaled by a factor $1/z$, where z reflects productivity. The condition determining the first-best production is $u'(q) = c'(q)/z$, giving $q^*(z)$ and, by equation (15), $i^*(z)$. By implicit differentiation, we obtain the following condition

$$\frac{\partial q^*(z)}{\partial z} = \frac{q^*(z)}{z}\left[\frac{c''(q^*(z))q^*(z)}{c'(q^*(z))} - \frac{u''(q^*(z))q^*(z)}{u'(q^*(z))}\right]^{-1},$$

which is strictly positive by the strict concavity of the utility function and the convexity of the cost function. The derivative of the optimal interest rate on $i^*(z)$ with respect to z is

$$\frac{\partial i^*(z)}{\partial z} = -\frac{i^*(z)}{z}\frac{u(q^*(z))}{c(q^*(z))/z}\frac{u'(q^*(z))q^*(z)}{u(q^*(z))}\frac{\partial q^*(z)}{\partial z}\frac{z}{q^*(z)},$$

which is strictly negative, since the elasticity of first-best output is strictly positive. Thus, the optimal interest payment on active balances is countercyclical.

3.2.2.2 Congestion Externality

Next, we consider the case in which the congestion externality prevails, with $\theta > \epsilon^*$, making participation excessive relative to efficiency. This scenario is examined by Lagos and Rocheteau (2005) in a cash economy. They find that if $\theta = 1$ and buyers have all the bargaining power, the optimal policy is the Friedman rule. The same result holds in our physical currency economy, as the two economies are identical if $\theta = 1$.[8] In what follows, we show that although the first best can no longer be achieved, digital currency achieves a strictly larger welfare compared to physical currency.

First, note that efficient participation in $q = q^*$ requires $F(q^*, i, i_p) = \epsilon^*/\theta < 1$, which, in turn, requires $i < 0$. However, if $i < 0$, $q = q^*$ violates the condition for the existence of a monetary equilibrium. Intuitively, if $q = q^*$ there is no liquidity premium. In this case, if $i < 0$ and idle balances pay a higher interest rate than active balances, buyers strictly prefer to keep their balances. In other words, q must be strictly lower than q^* if $i < 0$. Hence, when the congestion externality prevails,

[8] Lagos and Rocheteau (2004) generalize this result for arbitrary values of θ. They consider Nash bargaining, but it is straightforward to adapt their proof to the case of Kalai bargaining.

the scheme with interest payments on active and passive balances does not attain the first-best.

However, it is still possible to show that paying positive interest on passive balances and using τ to implement the Friedman rule strictly improves upon the policy $\tau = \beta - 1$ and $i = i_p = 0$. In particular, consider the policy $i = -\sigma i_p$ with $\sigma \in (0, 1)$ and $\tau = -(1-\beta)(1+i_p) - \alpha\mu(1/\alpha)\sigma i_p$. This policy satisfies (12) at equality, so it implements the Friedman rule in the digital currency economy. Denote by $(\alpha(i_p), q(i_p))$ the equilibrium allocation as a function of policy. Under this policy, the derivative of the welfare $W(i_p) = \alpha(i_p)\mu(1/\alpha(i_p))U(q(i_p)) - k(\alpha(i_p))$ with respect to i_p is

$$\frac{\partial W(i_p)}{\partial i_p} = \mu(1/\alpha)\left[\epsilon^* U(q) - \theta\frac{(1+i_p)\,U(q) - u(q)\sigma i_p}{1 + (1-\theta\sigma)\,i_p}\right]\frac{\partial \alpha}{\partial i_p} + U'(q)\frac{\partial q}{\partial i_p}.$$

If $i_p = 0$, we have $i_a = 0$ and $\tau = -(1-\beta)$. This intervention corresponds to the optimal policy in the physical currency economy. Evaluating the derivative of welfare at $i_p = 0$, we obtain

$$\frac{\partial W(0)}{\partial i_p} = (\theta - \epsilon^*)\frac{U(q^*)\left[(1-\theta)u(q^*) + \theta c(q^*)\right]\theta\sigma}{\frac{k'(\alpha)}{\alpha}\left[1 - \epsilon^* + \frac{k''(\alpha)\alpha}{k'(\alpha)}\right]},$$

where we used the fact that $q(0) = q^*$. The implication is that if $\theta > \epsilon^*$, $i_p = 0$ is not optimal. In particular, there is an improvement under digital currency away from $i_p = 0$ if we set $i_p > i_a > 0$. The next proposition follows.

Proposition 3 *If $\theta > \epsilon^*$, there exist policy schemes that improve upon the Friedman rule and zero interest payment that rewards passive balances with higher interest than active balances.*

In this case, paying higher interest on passive balances relative to the interest paid on active balances provides an improvement. This is because rewarding idle relative to active balances discourages participation that is excessive relative to efficiency.

3.2.3 Hosios Condition

We can summarize the results obtained so far as follows. As regards participation, two externalities are at work in this model, known in labor economics as the thick market and the congestion externality, respectively. Due to these externalities, traders do not correctly incorporate the effect of their participation decision on the equilibrium. When the thick market externality prevails, traders under participate; when the congestion externality prevails, traders over participate. Whether the former or the latter externality prevails depends on the bargaining power θ, which determines how trades split the day surplus, relative to the elasticity of the matching function in efficient participation ϵ^*, which determines the correct split of the surplus. In the knife-edge case in which the bargaining power coincides with the elasticity of the matching function evaluated at efficient participation, that is, the Hosios condition of labor economics $\theta = \epsilon^*$, the two externalities exactly offset each other and the first-best allocation for extensive and intensive margins is achieved by setting lump-sum taxation appropriately according to the well-known Friedman rule. For lower values of the bargaining power $\theta < \epsilon^*$, the thick market externality prevails, and, as a consequence, the equilibrium participation is too small relative to efficiency. In this case, paying higher interest on active than passive balances, something that can be done with digital but not with physical currency, increases the day trade surplus, providing an additional incentive to participate in day trade. Setting the net interest rate and lump-sum taxation appropriately, the first-best can be restored in the digital currency economy. For higher values of the bargaining power $\theta > \epsilon^*$, congestion externality prevails and, as a consequence, equilibrium participation is too large relative to efficiency. In this case, participation should be discouraged by rewarding passive balances relatively more than active balances. Although full efficiency cannot be obtained in this case, the interest payments available with digital currency can help improve the allocation relative to the optimal policy benchmark with physical currency.

3.2.4 Velocity of Circulation

It is well known from the work of Levine (1991), Kehoe, Levine, and Woodford (1992) and Wallace (2014) that extensive margins play a key role in monetary economies. In our framework, the extensive margin is captured by the endogenous participation decision of the buyers. Observing the flows of digital currency allows the implementation of intervention schemes that improve welfare by giving additional incentives to participate in trade, thus increasing the velocity of money. With both physical and digital currency, a fraction α of traders end up at meetings in which the currency changes hands for sure, with unit velocity of circulation, and a fraction $1-\alpha$ of traders keep their currency in the account, in which case the velocity is nil. Therefore, the velocity of circulation in both economies is α. If $\theta < \epsilon^*$, the velocity of circulation is inefficiently low in equilibrium relative to efficiency since the thick market externality prevails. Stimulating the velocity by paying a premium for active balances relative to idle ones helps improve the allocation. Under the optimal policy, the first-best is achieved. If $\theta > \epsilon^*$, the velocity of circulation of currency needs to be discouraged, being too large at equilibrium relative to efficiency, since the congestion externality prevails. Reducing the velocity by paying a premium for holding idle balances relative to active ones helps improve the allocation.

3.2.5 Friedman Rule

With physical currency, when the only policy instrument is τ, the optimal monetary policy is the Friedman rule. There is no role for raising the currency growth rate above the Friedman rule even when the Hosios condition does not hold and the search intensity is inefficiently high or low, since the currency growth rate affects search intensity through the match surplus, which is maximal at the Friedman rule under Kalai bargaining.

However, if the extensive margin was captured as a choice by agents to participate in the decentralized market as buyer or seller (e.g., Rocheteau and Wright (2009)), then inflation would have a first-order effect on the participation decision, even in the physical currency economy. In such a model, a deviation from the Friedman rule would increase welfare when congestion externality prevails (e.g., Nosal and Rocheteau (2011)). We have shown that, with digital currency, a deviation from the Friedman rule can be optimal if there is a congestion externality, but it can also be optimal when there is a thick market externality.

3.2.6 Complete Participation

Finally, consider what happens when Assumption 1 is violated, that is, $k'(1) \leq \mu(1)\epsilon(1)U(q^*)$. In this case, the first-best participation is always complete, $\alpha^* = 1$ and $\epsilon(1) = \epsilon^*$. Since efficient participation is complete, there cannot be excessive participation relative to efficiency; hence, the only relevant case is when the thick market externality prevails, with $\theta < \epsilon^*$. Define $\theta^* \equiv k'(1)/[\mu(1)U(q^*)]$. In this case, given τ, the interest rates are chosen to satisfy the inequality $\theta F(q^*, i, i_p) \geq \theta^*$. There are two cases. If $\theta \geq \theta^*$, since $F(q^*, 0, 0) = 1$, the first-best can be achieved without interest payments, $i = i_p = 0$. With zero interest payments, the optimal policy is $\tau = \beta - 1$. In this case, the first-best can be achieved in the physical currency economy by the Friedman rule. Next, consider $\theta < \theta^* < 1$. A natural policy candidate for the implementation of the first-best involves setting $i_p = 0$. The net interest payment satisfies the following condition

$$i \geq \frac{\theta^* - \theta}{\theta[\delta(q^*) - \theta^*]} \equiv i^{**},$$

where $i^{**} > 0$, since $\theta < \theta^* < 1$ and $\delta(q^*) > 1$. We take the smallest interest payment i^{**} as the natural policy choice. Using (13) and i^{**}, we obtain

$$\tau^{**} = \beta - 1 - \frac{\mu\,(1 - \beta\theta)\,(\theta^* - \theta)}{\theta[\delta(q^*) - \theta^*]}.$$

Since $i^{**} > 0$, $i \geq 0$ is satisfied. Moreover, $i^{**} > 0 = i_p^{**}$ implies $\phi > \beta\phi_{+1}$. The optimal policy is away from the Friedman rule. In fact, money is created under the optimal policy if $\tau^{**} + \mu i^{**} > 0$, that is, when the agents are sufficiently patient. Therefore, our results extent to the case in which efficiency requires complete participation.

4 Constrained Intervention

In this section, we compare our results with the literature by exploring constrained intervention schemes. First, we examine interventions that pay the same interest rate on idle and active balances. Second, we examine interventions that do not involve the taxation of balances. For simplicity, from now on we assume a degenerate matching function with unit elasticity.

4.1 Uniform Interest on Balances

The existing literature distinguishes sovereign digital currency and cash assuming that they are imperfect substitutes as means of payment. In particular, sovereign digital currency is a closer substitute for bank-issued debt than cash. Except for this difference, sovereign digital currency is treated as a digital form of cash and the only notable difference between them is the fact that uniform interest can be paid on sovereign digital currency.

In our model, treating digital currency as physical currency in digital form means assuming that the former embeds the same record-keeping technology as the latter, which implies that the monetary authority can no longer distinguish between passive and active balances and must set $i_a = i_p$, that is, $i = 0$. Setting $i = 0$ in (11), we obtain a condition whose only difference from the equilibrium condition in the physical currency economy (7) is that in the digital currency economy with uniform interest, the policy instrument is given by τ divided by the gross nominal interest rate. Consider now the extensive margin of the digital currency economy. If $i = 0$ in (9), we obtain a condition which coincides with (5), the extensive margin in the physical currency economy. The next proposition follows.

Proposition 4 *With $i = 0$, an outcome (α, q) is an equilibrium in the physical currency economy if and only if it is an equilibrium in the digital currency economy.*

Therefore, assuming that digital currency is simply a digital form of physical currency, the differences between these instruments must be related to extrinsic features of the economy that make physical and digital currencies imperfect substitutes as means of payments rather than intrinsic features.

4.2 No Taxation of Balances

The taxation of balances may not be feasible in pure currency economies. The idea is that the same frictions on commitment and monitoring that make money essential also prevent the working of lump-sum taxation schemes.[9] In our case, this translates into a restriction on policies with $\tau \geq 0$. Even if balances cannot be taxed, there still exists an open region of parameters that replicates the results obtained in the previous section, that is, the observation of balance flows still allows digital currency to dominate physical currency and implement the first-best.

To see the point, assume that the monetary authority transfers $\tau(\phi m) \geq 0$ real balances to each account with ϕm real balances at the end of the day market for all

[9] This has been pointed out, among others, by Hu, Kennan, and Wallace (2009), Andolfatto (2010), Wallace (2014), and Bajaj et al. (2017)

$m \geq 0$.[10] Thus, if buyers give ϕm real balances to sellers with zero balances during the day market, sellers anticipate that they will receive a transfer of $\tau(\phi m)$ real balances at the end of the day market. Under Kalai bargaining, the quantity produced in the meetings satisfies

$$\phi m + \tau(\phi m) = (1-\theta)u(q) + \theta c(q) \equiv g(q). \tag{17}$$

In the physical currency economy, consider a transfer scheme where the monetary authority makes a transfer of real balances to each account that conditions on the real balance in the account. The optimal scheme compensates for the inflation tax accounts with real balances larger than or equal to the socially optimal amount, that is, the amount of real balances that induces the sellers to produce the efficient quantity. The transfer is $\tau g(q^*)/(1+\tau)$ if $\phi m \geq g(q^*)/(1+\tau)$ and nil otherwise. Provided τ is large enough and traders are patient enough, this scheme ensures that efficient quantities are produced. In particular, the transfer scheme replicates the allocation achieved under the optimal policy in the physical currency economy with taxation. However, since this transfer scheme cannot separate idle and active balances, it does not directly give buyers the incentive to participate. Hence, also without taxation, participation remains inefficiently low in the physical currency economy.

Consider the digital currency economy. Since flows in and out of accounts are observed by the authority, the optimal scheme compensates for the inflation tax only active balances. This ensures not only efficient production at trade meetings but also efficient participation of buyers. In particular, the transfer scheme gives a positive transfer $\tau(\phi m) = \tau g(q^*)/(1+\tau)$ exclusively to active balances that are at least the socially optimal amount $\phi m \geq g(q^*)/(1+\tau)$. This is the case of the balances held by sellers who participated in a trade meeting in which the buyers brought the socially optimal amount of real balances. Sellers reciprocate by producing the efficient quantity in exchange for these balances. Buyers who brought $g(q^*)/(1+\tau)$ real balances choose α to maximize $\alpha u(q^*) + (1-\alpha)g(q^*)/(1+\tau) - k(\alpha)$. Ignoring the corner solution, the optimum satisfies $u(q^*) - g(q^*)/(1+\tau) = k'(\alpha)$. In the night

[10] The transfer scheme we use is adapted from Bajaj et al. (2017). This transfer scheme is feasible because balances can be observed in a physical currency economy. In Bajaj et al. (2017) transfers are given to agents at the beginning of the day market, while we are considering a slightly modified version, where agents receive their transfers at the end of the day market. This is immaterial in the physical currency economy but it matters in the digital currency economy.

market, there are two possibilities. If buyers bring $\phi m < g(q^*)/(1+\tau)$, their payoff is

$$\underline{V} = -\phi_{-1}m + \beta\left[\alpha u(q) + (1-\alpha)g(q) - k(\alpha)\right],$$

where $\alpha = k'^{-1}(\theta U(q))$. If, instead, buyers bring $g(q^*)/(1+\tau)$ real balances, their payoff is

$$\overline{V} = -g(q^*) + \beta\left[u(q^*) - k(\alpha^*)\right].$$

In the appendix, we prove that if τ is sufficiently large and traders sufficiently patient, we have $\overline{V} \geq 0 \geq \underline{V}$.

Proposition 5 *There are lower bounds $\underline{\tau}$ and $\underline{\beta}$, such that if $\tau \geq \underline{\tau}$ and $\beta \geq \underline{\beta}$, the transfer scheme $\tau(\phi m)$ replicates the allocation achieved under the optimal policy in the digital currency economy with the taxation of balances.*

Proof. Since $\phi \to 0$ when $\tau \to \infty$, (17) implies that $q \to 0$ when $\tau \to \infty$. Thus, there exists $\underline{\tau}$ such that $\underline{V} \leq 0$ for all $\tau \geq \underline{\tau}$. If $\tau \geq \underline{\tau}$, a sufficient condition for buyers to bring real balances $g(q^*)/(1+\tau)$ is $\overline{V} \geq 0$, which can be rewritten as $\beta \geq g(q^*)/[g(q^*) + k'(\alpha^*) - k(\alpha^*)] \equiv \underline{\beta}$ where $\underline{\beta} < 1$ since $k'(\alpha^*) > k(\alpha)/\alpha^* \geq k(\alpha^*)$ by the properties of the function $k(\cdot)$ and $\alpha^* \leq 1$. ■

The only substantial difference between these schemes and the optimal interventions considered in the case where real balances can be taxed is that agents must be sufficiently patient, both in the physical currency and in the digital currency economy. This is so because, unlike the scenario where real balances can be taxed and the optimal intervention can condition on the agent's discount factor, here the injection of real balances must be sufficiently large to convert the decision of a buyer on how much real balances to bring, into a binary choice between bringing the socially optimal amount and bringing zero balances. Sufficient patience is required for the former option to dominate.

5 Robustness

In this section, we extend the result to a larger class of bargaining procedures than just Kalai bargaining, to a modified environment in which trade occurs simultaneously in decentralized and centralized markets, and show that the scheme with the payment of interest on active balances is robust to manipulation attempts that involve the opening of shadow accounts.

5.1 General Trading Mechanism

We have adopted the Kalai bargaining procedure for simplicity as it makes computations easier. Other bargaining schemes could be used without altering the main results. Next, we adapt to our setting the monetary mechanisms proposed by Gu and Wright (2016). We then show that our main result holds, that is, the observation of balance flows makes it possible to achieve first-best. Consider the following trading mechanism $\Gamma(\phi m) = (\Gamma_p(\phi m), \Gamma_q(\phi m))$, where $\Gamma_p(\phi m)$ sets the real balances transferred from buyers to sellers, and is given by

$$\Gamma_p(\phi m) = \begin{cases} \phi m & \text{if } \phi m < p^* \\ p^* & \text{if } \phi m \geq p^* \end{cases},$$

where p^* is defined as the minimum payment required for a buyer to receive q^* units of goods from the seller; while $\Gamma_q(\phi m)$ sets the quantity produced by the sellers and is given by

$$\Gamma_q(\phi m) = \begin{cases} v^{-1}((1+i)\phi m) & \text{if } \phi m < \frac{p^*}{1+i} \\ q^* & \text{if } \phi m \geq \frac{p^*}{1+i} \end{cases},$$

where $v(\cdot)$ is a strictly increasing, twice continuously differentiable function, with $v(0) = 0$ and $v(q^*) = \frac{p^*}{1+i}$. This coincides with the trading mechanism in Gu and Wright (2016) for $i = 0$. In the digital currency economy, we need to take into account that, if buyers transfer ϕm real balances to sellers during the day market, at the beginning of the night market, sellers will have $(1+i)\phi m$ real balances. Note that the terms of trade do not depend on the real balances of sellers, and the efficient quantity is produced if $(1+i)\phi m \geq p^*$. With Kalai bargaining, the function $v(\cdot)$ is given by

$$v(q) = \frac{(1+i)\left[(1-\theta)u(q) + \theta c(q)\right]}{1+\theta i};$$

with Nash bargaining, the function is given by

$$v(q) = \frac{(1+i)\left[(1-\theta)u(q)c'(q) + \theta c(q)u'(q)\right]}{\theta(1+i)u'(q) + (1-\theta)c'(q)};$$

and with competitive pricing, the function is $v(q) = pq$, where p is taken as given by the traders but is equal to $c'(q)$ in equilibrium, giving $v'(q) = c'(q)$.

Without loss of generality as far as efficient allocations are concerned, henceforth, we restrict attention to interventions that pay zero interest on passive balances. This implies that, in equilibrium, sellers do not bring balances, and buyers do not bring balances they are not planning to use. We start with the participation decision of buyers with real balances ϕm at the beginning of the day market. Buyers choose $\alpha \leq 1$ to maximize $\alpha u(q) + (1-\alpha)\phi m - k(\alpha)$. Ignoring the corner solution, the optimality condition for the participation choice is

$$u\left(q\right) - \frac{v(q)}{1+i} = k'(\alpha).$$

Note that, given q, an increase in i increases the incentive of buyers to participate in the trade. Consider now the night market. Buyers choose z to maximize $-\phi m + \beta[\alpha u(q) + (1-\alpha)\phi_{+1} m - k(\alpha)]$. The intertemporal condition for the optimum is

$$\frac{u'(q)}{v'(q)} = \frac{\tau + 1 - \beta + \alpha\beta + \alpha i}{\alpha\beta(1+i)}.$$

where we have used $M_{+1} = (1 + \tau + \alpha i)M$ and the stationarity of real balances. We now determine the optimal intervention. Under Assumption 1 with $\mu = \epsilon = 1$, first-best (α^*, q^*) satisfies $u'(q^*) = c'(q^*)$ and $u(q^*) - c(q^*) = k'(\alpha^*)$. To achieve first-best, we set (τ, i) so that

$$\frac{u'(q^*)}{v'(q^*)} = \frac{\tau + 1 - \beta + \alpha^*\beta + \alpha^* i}{\alpha^*\beta(1+i)}, \tag{18}$$

holds, together with

$$u\left(q^*\right) - \frac{v(q^*)}{1+i} = k'(\alpha^*).$$

If $i = 0$, efficient participation requires $v(q^*) = c(q^*)$, which is not generically satisfied. For example, if $v(q)$ is determined by Kalai or Nash bargaining, this is only possible if $\theta = 1$, that is, the buyer has full bargaining power. Thus, in general, the first-best cannot be implemented if the authority cannot distinguish between active and passive balances. If, instead, $i > 0$, we can achieve efficient participation by setting

$$i^* = \frac{v(q^*)}{c(q^*)} - 1,$$

which is positive if and only if $v(q^*) > c(q^*)$. For example, with Kalai and Nash bargaining, the optimal interest payment is

$$i^* = \frac{1-\theta}{\theta[\delta(q^*)-1]}$$

, which is positive if and only if $\theta < 1$. With competitive pricing, the optimal interest payment is $i^* = c'(q^*)q^*/c(q^*) - 1$, which is strictly positive if and only if the elasticity of the cost function at first-best is greater than 1, that is, when the cost function is strictly convex. Using (18), we determine the optimal τ, which is given by

$$\tau^* = -(1-\alpha^*)(1-\beta) - \alpha^* \frac{v(q^*)}{c(q^*)}\left[1 - \beta \frac{u'(q^*)}{v'(q^*)}\right].$$

The existence of a monetary equilibrium requires $\tau^* \geq -(1-\beta) + \alpha^* - \alpha^* v(q^*)/c(q^*)$. Therefore, the first-best can be implemented by the policy (τ^*, i^*) if and only if

$$\frac{c'(q^*)q^*}{c(q^*)} \geq \frac{v'(q^*)q^*}{v(q^*)}. \tag{19}$$

Define the elasticity of the function $h(x)$ with respect to x as $\eta_h(x) \equiv h'(x)x/h(x)$. Thus, we immediately have the following proposition.

Proposition 6 *If the function $v(q)$ satisfies $v(q^*) > c(q^*)$ and $\eta_c(q^*) \geq \eta_v(q^*)$, there exist policy schemes that implement the first-best rewarding active balances with positive interest and passive balances with zero interest.*

With Kalai bargaining, computing the derivative $v'(q)$ and evaluating it at first-best, condition (19) becomes $(1-\theta)u(q^*)/\delta(q^*) \geq 0$ that is always satisfied. With Nash bargaining, computing the derivative $v'(q)$ and defining the elasticity of $h'(x)$ as $\eta_{h'}(x) \equiv |h''(x)x/h'(x)|$, (19) is satisfied if $[\eta_{c'}(q^*) + \eta_{u'}(q^*)][\delta(q^*) - 1] \leq q^*$, which holds if the sum of the curvatures of the utility and cost functions is bounded above. With competitive pricing, the condition is automatically satisfied since the elasticity of the cost function is never less than unity, being strictly increasing, convex, and nil at zero production.

5.2 Alternating Markets

The alternating market structure with decentralized and centralized trade characteristic of Lagos and Wright (2005) makes it especially easy to separate active and idle balances. One may wonder if our result would survive in an environment in which such separation is not so easy to accomplish.

Consider, for instance, the following variation of the baseline model.[11] Suppose that some agents trade in the decentralized market in odd periods and in the centralized market in even periods, while others trade in the decentralized market in even periods and in the centralized market in odd periods. In this set up, the task of separating active and passive balances is more difficult: both buyers entering the centralized market without money and leaving it with money, and sellers entering the decentralized market without money and leaving it with money have positive inflows into their accounts. As a result, if the monetary authority maintains the same policy used in the baseline model, it pays interest to active accounts in the centralized market, which have no bearing on the incentives of agents to participate in the decentralized market. However, in this variation of the baseline model, this issue can be solved if the authority considers the balance information of the last two periods. In fact, if the authority observes at the beginning of the period t that there was an inflow into the account in period $t-1$, and the account had no balances at the beginning of the period $t-2$, then this account belongs to a seller who participated in the decentralized market in period $t-1$, and not a buyer who participated in the centralized market in period $t-1$. This is so because, if a buyer participates in the centralized market in period $t-1$, it must have participated in the decentralized market in period $t-2$, and buyers always bring a positive balance to the decentralized market.

More generally, in settings with more complicated trading structures, the entire histories of monetary flows can be used to stimulate participation. The idea would be to exploit the higher correlation between sequences of monetary flows and participation decisions relative to the correlation between spot monetary flows and participation decisions.

[11] We thank an anonymous referee for suggesting this thought experiment.

5.3 Manipulation

Agents could try to manipulate the system to obtain undue interest on idle balances. If agents were allowed to open additional accounts, they could use them to transfer unused balances between accounts and receive interest payments on those balances. So far, we have avoided this possibility by assuming that each agent can have only one account with the monetary authority. Next, we introduce the possibility of manipulation by assuming that agents can open shadow accounts that the authority cannot associate with the owner of any existing account. However, in order to open a shadow account, agents have to incur a cost $\kappa > 0$ during the night market. The account can then be used in the next day market to transfer balances to receive interest i_a.

Consider the incentive for a buyer to open a shadow account at a cost κ. With the unit elastic matching function, the optimal policy is given by $i_p^* = 0$, $i_a = i^* > 0$ given by (15) and $\tau = \tau^*$ given by (16). Such a buyer holding an amount $\tilde{m}$ of currency chooses $\tilde{q}$ and $\tilde{d} \leq \tilde{m}$ to maximize $u(\tilde{q}) - \phi(1+i^*)\tilde{d}$ subject to the constraint $(1-\theta)\tilde{S}_b = \theta\tilde{S}_s$, where $\tilde{S}_b = u(\tilde{q}) - \phi(1+i^*)\tilde{d}$ and $\tilde{S}_s = -c(\tilde{q}) + \phi(1+i^*)\tilde{d}$. Note that the buyer receives interest i^* on passive balances. This allows us to rewrite the constraint $(1-\theta)\tilde{S}_b = \theta\tilde{S}_s$ as $\phi\tilde{m} = g(\tilde{q})/(1+i^*)$. Consider now the participation decision. Optimization leads to the optimality condition for participation

$$\theta U(\tilde{q}) = k'(\tilde{\alpha}). \tag{20}$$

Moving to the night market, the intertemporal condition for the optimum can be written as

$$\frac{u'(\tilde{q})}{g'(\tilde{q})} = 1 - \frac{1-\alpha^*\theta}{\tilde{\alpha}} \frac{1-\theta}{\theta[\delta(q^*)-1]+1-\theta} \equiv H(\theta). \tag{21}$$

The equations (20) and (21) jointly determine $(\tilde{\alpha}(\theta), \tilde{q}(\theta))$. If buyers choose not to open the account, the participation is given by (9) and the intertemporal condition is given by (11), both evaluated in the optimal policy. The net benefit of not opening the account is $\beta[\alpha^* u(q^*) - k(\alpha^*) - g(q^*)]$, while the net benefit of opening an additional account is $-\kappa + \beta[\tilde{\alpha}(\theta)u(\tilde{q}(\theta)) - k(\tilde{\alpha}(\theta)) - H(\theta)g(\tilde{q}(\theta))]$. Therefore, the condition that ensures that buyers have no incentive to open an additional account is

$$\beta[\alpha^* u(q^*) - k(\alpha^*) - g(q^*) - \tilde{\alpha}(\theta)u(\tilde{q}(\theta)) + k(\tilde{\alpha}(\theta)) + H(\theta)g(\tilde{q}(\theta))] + \kappa \geq 0. \tag{22}$$

Notice that the function $H(\theta) \leq 1$ with $H(1) = 1$. Since the allocation for a deviating buyer has $\tilde{q}(1) = q^*$ and $\tilde{\alpha}(1) = \alpha^*$, we see that the LHS of (22) is equal to $\kappa > 0$ for $\theta = 1$. By continuity, there is an interval of values of θ close to, but strictly smaller than 1 such that the incentive condition is still satisfied. To ensure that agents do not create shadow accounts even if they do not want to participate in the trade, $(1 + \tau^*)(1 + \kappa) \geq \beta(1 + i^*)$ is required, which is strictly satisfied for $\theta = 1$. By continuity, there exist values of θ close to, but strictly smaller than 1 such that this incentive condition is also satisfied. The next proposition follows.

Proposition 7 *Suppose that the system can be manipulated at a cost $\kappa > 0$. There exists an open interval of values of θ in which the optimal policy with digital currency $(\tau^*, i^*, 0)$ is incentive compatible.*

The optimal policy that reproduces the Friedman rule and achieves the best allocation attainable with physical currency is always available. Hence, for any positive cost of manipulation, the optimal policy with digital currency can always at least replicate the outcome with physical currency and, in an open set of economies, strictly improve upon it. Intuitively, this is achieved when the interest payments that induce efficiency can be kept small. If it is even slightly costly to manipulate the system, there are robust circumstances in which the extra information provided by the digital technology can be exploited to achieve efficiency while discouraging manipulation.

In principle, agents could try to manipulate the system by forming partnerships with other agents to transfer balances between accounts and get undue interest payments. However, in our economy, by assumption, agents meet randomly and bilaterally and cannot commit to future actions. These frictions, which are key to generating an essential role for currency as a medium of exchange, prevent the formation of partnerships of this type, as well as market trade and group interactions in general. In fact, if traders could establish long-term trading relations with each other, good allocations could be supported by reputation, as in repeated games. This would make the currency redundant as a trading instrument. Since we consider the essentiality of currency a key requirement in a fundamental theory of monetary trade, we exclude these types of partnership. Therefore, the only feasible manipulation schemes are individual.

6 Conclusion

We have argued that the key difference between digital currency and cash is in the technological ability to trace currency flows. Since additional information collected with digital technology can be ignored, any allocation achieved with cash can also be achieved with digital currency. However, if additional information is used to affect extensive margins, there are robust circumstances in which digital currency can help improve the best allocation that would be achieved with cash. The availability of this information makes the difference in the design of optimal policy.[12]

When the thick market externality prevails, the payment of interest on active balances outperforms other reward schemes that have been identified as welfare improving in physical currency economies, such as the payment of interest on idle balances. This occurs because the positive interest differential between active and idle balances stimulates participation through its effect on the extensive margin, while rewarding idle balances affects the intensive, but not the extensive margin.[13] When the congestion externality prevails, rewarding idle balances more than active balances helps discourage participation. In economies with extensive margin distortions, digital currency can help restore or at least approach efficiency in a robust set of circumstances.

In our setting, efficient allocations can be achieved without banks. However, this does not mean that the adoption of sovereign digital currency would necessarily lead to disintermediation. In contrast, banks could play a key role in the actual implementation of the system, as digital payments could continue to work exactly as they do with the interest on active balances being paid through the banking system.

[12]In systems theory à la Wiener and Shannon, information is defined as "the difference that makes a difference", e.g. Bateson (1972), a definition we alluded to playfully in the title.

[13]As in Berentsen, Camera and Waller (2007), Ferraris and Watanabe (2008) and Geromichalos and Herrenbrueck (2016, 2017).

7 Appendix

7.1 Existence of the Digital Currency Equilibrium

We show that there are values $\underline{\theta} < \overline{\theta} \leq 1$ such that if $\theta \in [\underline{\theta}, \overline{\theta}]$, a digital currency equilibrium exists. The proof runs as follows. Define $\gamma \equiv (\tau, i, i_p)$ and $u'(q)/g'(q) \equiv f(q)$. The function $f(q)$ is continuous in q with $f(\infty) = 0$, $f(0) = (1-\theta)^{-1}$ and $f'(q) < 0$ by the properties of the fundamentals. Since $G(\alpha, \gamma) > 0$, there exists a unique $\tilde{q} \in (0, \infty)$ that satisfies (11) for any (α, γ) provided $G(\alpha, \gamma) \leq (1-\theta)^{-1}$. Since $G(\alpha, \gamma) < \infty$ is greater than unity for any θ and strictly decreasing in θ while $(1-\theta)^{-1} \in [1, \infty)$ is strictly increasing in θ, there is a unique value $\underline{\theta} \in (0, 1)$ such that for $\theta \geq \underline{\theta}$ the inequality is satisfied. Define $q = f^{-1}(G(\alpha, \gamma)) \equiv \varphi(\alpha)$ and plug it into (9), obtaining the function $\Phi(\alpha) \equiv \mu(1/\alpha)\theta U(\varphi(\alpha))F(\varphi(\alpha), i, i_p) - k'(\alpha)$, which is continuous in α, with $\Phi(0) > 0$, by the properties of the fundamentals; $\Phi(1) \leq 0$, if $\epsilon(1) \geq \theta F(\varphi(1), i, i_p)$ under Assumption 1. Notice that this gives an upper bound $\overline{\theta}$ that can be made to be compatible with $\underline{\theta}$ rescaling $\epsilon(1)$ appropriately. By the intermediate value theorem, there exists $\tilde{\alpha} \in (0, 1]$ that satisfies $\Phi(\alpha) = 0$.

7.2 Walrasian Equilibrium

Consider the physical currency economy without trading externality. Replace the frictional decentralized market with a centralized Walrasian market, interpreting the buyers' search intensity as the probability of participation. Let q denote the quantity consumed by buyers during the day. Welfare is $\alpha u(q) - c(\alpha q) - k(\alpha)$. The efficient allocation $(\hat{\alpha}, \hat{q})$ solves $u(\hat{q}) - c'(\hat{\alpha}\hat{q})\hat{q} = k'(\hat{\alpha})$, and $\hat{\alpha}[u'(\hat{q}) - c'(\hat{\alpha}\hat{q})] = 0$. Let p denote the price of day good in units of the night good. During the day, sellers choose q_s to maximize $pq_s - c(q_s)$, which gives $p = c'(q_s)$. Buyers with m units of money choose α to maximize $\alpha u(\phi m/p) + (1-\alpha)\phi m - k(\alpha)$, which gives $u(\phi m/p) - \phi m = k'(\alpha)$. In the previous night, buyers chose m to maximize $-\phi m + \beta\{-k(\alpha) + \alpha u(\phi_{+1} m/p) + (1-\alpha)\,\phi_{+1} m\}$, whose solution is $\phi = \beta\phi_{+1}[1 - \alpha + \alpha u'(q)/p]$, which, using $p = c'(q_s) = c'(\alpha q)$ and the stationarity of the real balances, can be written as $(1+\tau)/\beta = \alpha u'(q)/c'(\alpha q) + 1 - \alpha$. Using $p = \phi m/q$, the optimal buyer search intensity is $u\,(q) - c'(\alpha q)q = k'(\alpha)$. The Friedman rule, $\tau = \beta - 1$, achieves the first-best.

References

[1] Andolfatto David (2010), Essential Interest-bearing Money, *Journal of Economic Theory* 145, 1495-1507

[2] Andolfatto, David (2021), Assessing the Impact of Central Bank Digital Currency on Private Banks, *Economic Journal* 131, 525-540

[3] Aruoba, Boragan, Rocheteau, Guillaume, and Chris Waller (2007), Bargaining and the Value of Money, *Journal of Monetary Economics*, 54, 2636-2655

[4] Bajaj Ayushi, Tai-Wei Hu, Guillaume Rocheteau and Mario Rafael Silva (2017), Decentralizing constrained-efficient allocations in the Lagos–Wright pure currency economy, *Journal of Economic Theory*, 167, 1-13

[5] Bateson, Gregory (1972), *Steps to an Ecology of Mind*, University of Chicago Press

[6] Berentsen Alex, Camera Gabriele and Chris Waller (2007), Money, Credit and Banking, *Journal of Economic Theory*, 135, 171-195

[7] Chiu Jonathan, Davoodalhosseini Seyed Mohammadreza, Jiang Janet and Yu Zhu (2023), Bank Market Power and Central Bank Digital Currency: Theory and Quantitative Assessment. *Journal of Political Economy*, forthcoming

[8] Chiu Jonathan and Tsz-Nga Wong (2022), Payments on Digital Platforms: Resiliency, Interoperability and Welfare, *Journal of Economic Dynamics and Control*, 142, 1041-1073

[9] Ferraris Leo and Makoto Watanabe (2008), Collateral Secured Loans in a Monetary Economy, *Journal of Economic Theory*, 143, 405-424

[10] Geromichalos, Athanasios and Lucas Herrenbrueck (2016), Monetary Policy, Asset Prices, and Liquidity in over-the-counter Markets, *Journal of Money, Credit and Banking,* 48, 35–79

[11] Geromichalos Athanasios and Lucas Herrenbrueck (2017), A tractable Model of Indirect Asset Liquidity, *Journal of Economic Theory*, 168, 252-260

[12] Gu, Chao and Randall Wright (2016), Monetary mechanisms, *Journal of Economic Theory*, 163, 644-657

[13] Hu Tai-Wei, John Kennan, and Neil Wallace (2009), Coalition-Proof Trade and the Friedman Rule in the Lagos-Wright Model, *Journal of Political Economy*, 117, 116-137

[14] Hu, Tai-Wei and Guillaume Rocheteau (2020), Bargaining under Liquidity Constraints: Unified Strategic Foundations of the Nash and Kalai Solutions, *Journal of Economic Theory*, 189, 105098

[15] Hu, Tai-Wei and Kathy Zhang (2019), Responding to the Inflation Tax, Macroeconomic Dynamics, 23, 2378-2408

[16] Kalai Ehud (1977), Proportional solutions to bargaining situations: interpersonal utility comparisons, *Econometrica*, 45, 1623-1630

[17] Kehoe Tim, Levine David and Mike Woodford (1992), The Optimum Quantity of Money Revisited, in Partha Dasgupta, Douglas Gale, Oliver Hart, and Eric Maskin (eds.), *The Economic Analysis of Markets and Games: Essays in Honor of Frank Hahn*, Cambridge, MA: MIT Press, 501–526

[18] Keister Todd and Daniel Sanches (2023), Should Central Banks Issue Digital Currency? *Review of Economic Studies*, forthcoming

[19] Kocherlakota Narayana (1998), Money is Memory, *Journal of Economic Theory*, 81, 232-251

[20] Lagos Ricardo and Guillaume Rocheteau (2005), Inflation, Output and Welfare, *International Economic Review*, 46, 495-522

[21] Lagos Ricardo and Randall Wright (2005), A Unified Framework for Monetary Theory and Policy Analysis, *Journal of Political Economy*, 113, 463-484

[22] Levine David (1991), Asset Trading Mechanisms and Expansionary Policy, *Journal of Economic Theory*, 54, 148-164

[23] Li, Victor (1994), Inventory Accumulation in a Search-based Monetary Economy, *Journal of Monetary Economics*, 34, 511-536

[24] Li, Victor (1995), The Optimal Taxation of Fiat Money in Search Equilibrium, *International Economic Review*, 36, 927-942

[25] Liu Lucy, Liang Wang and Randall Wright (2011), On the Hot Potato Effect of Inflation: Intensive vs Extensive Margins, *Macroeconomic Dynamics*, 15, 191-216

[26] Nosal, Ed and Guillaume Rocheteau (2011), *Money, Payments and Liquidity*, MIT Press

[27] Rocheteau Guillaume and Ranadall Wright (2005), Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium, *Econometrica*, 73, 175-202

[28] Rocheteau Guillaume and Randall Wright (2009), Inflation and Welfare in Models with Trading Frictions, in *Monetary Policy in Low Inflation Economies*, Ed Nosal and Dave Altig eds, Cambridge University Press

[29] Wallace Neil (2014), Optimal Money-creation in 'Pure-Currency' Economies: a Conjecture, *Quarterly Journal of Economics*, 129, 259-274

[30] Williamson Sthephen (2022), Central Bank Digital Currency: Welfare and Policy Implications, *Journal of Political Economy*, 130, 2829-2861