ON BARGAINING NORMS AS SOLUTIONS TO COST–MINIMIZATION PROBLEMS

TYMON TATUR[†]

Abstract. This paper studies bargaining outcomes in economies in which agents may be able to impose outcomes that deviate from the relevant social norms but incur costs when they do so. It characterizes bargaining outcomes that are easiest for a society to sustain as part of a social norm which everybody will want to follow. Depending on technological assumptions, the approach yields the Nash bargaining solution, the Kalai-Smorodinsky solution, the equal monetary split, and other bargaining solutions. Set-valued solution concepts are derived that are relevant if one is unable or unwilling to make specific technological assumptions.

1. Introduction

Following Nash (1950, 1953) much of the modern theory on bargaining can be split into two branches. The axiomatic approach, born with Nash (1950), predicts bargaining outcomes based on assumptions about how outcomes in different bargaining situations differ. The strategic approach, also conceived by Nash (1953), considers a single bargaining situation in isolation and uses a non-cooperative game to model the strategic incentives players may face when negotiating an agreement. Once an agreement is reached, the game typically ends. In contrast, this paper is concerned with bargaining outcomes in societies where cooperation is governed by social norms, including ethical and moral norms. The idea that social norms may be relevant in

[†] Department of Economics, University of Bonn, Adenauerallee 24-42, D-53113 Bonn; Email: tatur@uni-bonn.de. I thank Philip Strack, Faruk Gul, Stephen Morris, Antonio Penta, Wolfgang Pesendorfer, Juuso Valimaki, Francoise Forges, Marek Pycia, Stephan Lauermann, Benny Moldovanu, Larry Samuelson, Paul Heidhues, and three anonymous referees for their helpful questions and comments. I am also grateful to Jonas Neuschäfer for pointing out a number of typos and minor mistakes in an earlier draft of the paper.

bargaining is not new. For instance, Kenneth Arrow (1971, p. 22) argued that a primary reason for the existence of social norms may be to facilitate cooperation by creating environments where individuals can trust each other, thereby making more efficient cooperation possible:

> "It is a mistake to limit collective action to state action... I want to [call] attention to a less visible form of social action: norms of social behavior, including ethical, and moral ones. I suggest as one possible interpretation that they are reactions of society to compensate for market failure. It is useful for individuals to have some trust in each other's word. In the absence of trust, it would become very costly to arrange for alternative sanctions and guarantees, and many opportunities for mutual beneficial cooperation would have to be forgone."

What allocation of surplus should we expect to be imposed as part of a social norm if such norms are used to avoid inefficiencies that would occur in the absence of such norms?

To understand the basic idea of the approach proposed in this paper, consider the following stylized example. Two agents can collaborate to costlessly produce a good that is worth \$1. There is bargaining over how to divide the created \$1 surplus. In other words, agents can agree on any allocation of surplus $(x_1, x_2) \in \{[0,1]^2 : x_1 + x_2 = 1\}$, where x_1 is the surplus received by player 1 and x_2 the surplus received by player 2. However, in the spirit of the inefficiencies mentioned by Arrow, assume such agreements are not easily enforceable. Before the cooperation is complete and the final product can be sold, each side will repeatedly have an opportunity to "steal" the unfinished good, which can be sold for \$0.60. Note that no matter what allocation of surplus the two agents agree on, in the absence of ethical or other norms at least one party will have an incentive to break the agreement if doing so guarantees them \$0.60. Moreover, if an agent expects the other agent to steal the unfinished product at the first opportunity – leaving them with nothing – that agent would have an incentive to steal the unfinished good themself before the other agent does. Thus, we can expect that each agent will try to "steal" the unfinished good if given the chance. Hence, the good will never be completed and the surplus divided between the two agents will be at most \$0.60 instead of \$1. This is inefficient in the sense that, if both agents could trust each other's assurances that they will complete the project, they could create a surplus of \$1 instead and divide it in such a way that each of them would be strictly better off. Perhaps it was, among others, this sort of situation that Arrow had in mind when he wrote that "it is useful for individuals to have some trust" and "in the absence of trust" it may be the case that "many opportunities for mutual beneficial cooperation would have to be forgone."[1]

Imagine that a social norm is in place mandating that the project be completed when agreed upon – where stealing the unfinished product would violate the norm – and after the product is finished player 1 receives x_1 and player 2 receives x_2 dollars, where x_1 and x_2 are non-negative numbers that add up to one. Furthermore, suppose that if one player violates the norm that player will incur a "deviation cost" of m dollars. In the case of an internalized norm, m could capture costs that the agent needs to incur in order to overcome the anxiety after breaking the norm. If observed deviations from the norm result in sanctions, m could represent the costs the agent needs to incur to avoid those sanctions or the disutility experienced when facing those sanctions. It is clear that if m is sufficiently large – in the considered example at least \$0.60 – any allocation x can be sustained as a norm. While any allocation of surplus can be sustained as a norm if the "punishment" m is sufficiently large, the minimal m needed to sustain trust will depend on the allocation. Indeed, if player i gets x_i under the allocation x, then in order for them to not be willing to break the norm

[1]The example illustrates why inefficiencies may arise in prolonged collaborations when trust is absent. It is perhaps less obvious that norms may also prevent inefficiencies in situations where complete contracts specifying any division of surplus are available. Consider, for instance, a buyer and a seller who can draw up a legally binding contract specifying the terms of delivery and the price for the goods sold, knowing that courts will enforce both the delivery and payment of the goods if necessary. As was pointed out by Crawford (1982), in said situations inefficiencies will often occur if agents can imperfectly commit to bargaining positions before bargaining starts. Social norms can also help eliminate such inefficiencies – if deviations from the norm result in sanctions or other costs, incentives to imperfectly commit to a bargaining position in an attempt to get a division more favorable than the one specified by the norm will decrease.

and leave with \$ 0.60 it must be that $0.60 - m \leq x_i$. Thus, for any allocation x, the set of "punishments" m for which x can be sustained as a norm is given by

$$\mathcal{S}(x) = \{m \in [0, \infty) : m \geq 0.60 - \min(x_1, x_2)\}.$$

In other words, if we identify norms with pairs (x, m) where x is the allocation of surplus postulated by the norm and m specifies how deviators are sanctioned, then the norms that can be sustained are exactly those norms (x, m) for which $m \in \mathcal{S}(x)$. If such a norm is used, which will it be?

Sustaining a social norm that prevents individuals from stealing the unfinished good will, of course, involve certain social costs. For instance, if the norm is sustained by punishing deviators, potential deviators need to be monitored and the individuals punishing deviators need to be incentivized. Imagine that $\kappa(m)$ is the minimal cost that society needs to incur to sustain a norm with an enforcement technology in which deviators incur a cost of m. It appears natural to assume that $\kappa(m)$ increases in m. Consider now the problem of finding the cheapest sustainable norm (x, m) among all the norms that can be sustained, namely the problem

$$\min_{x} \min_{m \in \mathcal{S}(x)} \kappa(m). \tag{1}$$

In the example considered above, this problem is easy to solve. Since the minimal punishments needed to sustain an allocation x are given by $0.60 - \min(x_1, x_2)$, it is clear that among all allocations x that are sustainable as part of a norm there is a unique one that is the cheapest to maintain, and that is the allocation x^{even} in which each player receives \$0.50, that is where the dollar is split evenly. It is no coincidence that this allocation x^{even} also has the property that for any $x' \neq x^{\text{even}}$ it is the case that $\mathcal{S}(x') \subsetneq \mathcal{S}(x^{\text{even}})$, namely that the set of punishments with which x^{even} can be sustained is strictly larger than the set of punishments with which any other allocation x' can be sustained.

In this paper, we will analyze the allocation of surplus in bargaining problems by asking which allocation of surplus is least costly for society to sustain as a social norm.

We will see that in the case of two-player bargaining under complete information[2] this approach yields different predictions depending on the nature of the assumed punishments. In particular, standard solution concepts like the Kalai-Smorodinsky solution, the Nash bargaining solution, and the equal monetary split, can all be understood as unique solutions to our cost-minimization problem for natural punishment technologies. We also show how our approach can be used to yield set valued solution concepts that generalize the bargaining solutions mentioned above and are useful if little is known about the way in which norms are enforced and about the function κ.

The paper is organized as follows. Section 2 introduces the model and basic concepts. Section 3 considers several examples and, in particular, shows how for appropriate norm enforcement technologies, our approach yields the Kalai-Smorodinsky solution and the Nash bargaining solution. Section 4 derives some more general results. The main results here are Theorems 1 and Theorem 2 that characterize all allocations that are the unique solution to some cost-minimization problem and, at the same time, characterize all allocations that are the easiest to sustain for some enforcement technology. Section 5 discusses extensions. Section 6 concludes.

Since this paper provides alternative foundations for concepts like the Nash bargaining solution, the Kalai-Smorodinsky solution, and more, our work can be seen as part of a large body of literature discussing foundations for those and related concepts. Our approach, however, differs from typical papers using the axiomatic approach (e.g., Nash (1950); Kalai-Smorodinsky (1975); or Rubinstein et al. (1992)), as a single type of bargaining problem is considered in isolation and no assumptions are made about how bargaining outcomes will change if some aspects of the bargaining situation—like the set of alternatives or the preferences of the players—are modified. Our approach also differs from papers using the strategic approach (e.g., Nash (1953); Rubinstein (1982); Abreu & Gul (2000); Compte & Jehiel (2010); Perry & Reny (1994)) and, more generally, papers using non-cooperative game theory, given that we do not select outcomes based on standard solution concepts used in non-cooperative game theory.

[2] A companion paper studies bargaining among more players.

If one thinks about social norms that are internalized (i.e., part of the agent's preferences) the proposed approach seems related to a literature studying the evolution of preferences in reduced models in which nature designs preferences to avoid certain inefficiencies, such as in Samuelson (2004) or Samuelson and Swinkels (2006). Papers that use evolutionary game theory to select Nash equilibria in non-cooperative bargaining games (e.g., Young (1993)) appear less related because the methodology is again very different.

2. Model

Consider the problem of two agents who can engage in some activity that creates a monetary surplus. For the sake of concreteness we will assume that the surplus is equal to \$1.

Let

$$\mathcal{X} = \{(x_1, x_2) \in [0,1]^2 : x_1 + x_2 = 1\}$$

be the set of possible efficient allocations of the monetary surplus, where $(x_1, x_2) \in \mathcal{X}$ is interpreted as an allocation where player 1 receives x_1 and player 2 receives x_2. If an allocation of surplus $x \in \mathcal{X}$ is implemented, players receive von Neumann-Morgenstern utilities $u_1(x_1)$ and $u_2(x_2)$, respectively, where u_i for $i = 1, 2$ are differentiable utility functions satisfying $u_i' > 0$ and $u_i'' \leq 0$. In the following we will assume that players are symmetric in all aspects except their utility functions u_i.[3]

In the example considered in the introduction, we assumed that during the cooperation agents have the option to "steal" an unfinished product that can be sold for \$0.60. We then considered social norms that we identified with pairs (x, m), where x was the allocation of surplus postulated by the norm and m was a real number describing the consequences an agent had to face when deviating from the norm.

In this section, we will consider a more general framework and, in particular, allow the agents to attempt to "grab" any fraction of the produced surplus. We will also allow for more subtle enforcement mechanisms where the consequences that an agent

[3]See Section 5.1 for some remarks about the case where players are asymmetric in other aspects.

faces after a deviation can, among other things, depend on how far they deviated from the social norm in place. While we will still be able to identify social norms with pairs (x, c), where x is the allocation of surplus postulated by the norm and c describes the consequences agents have to face after a deviation, the parameter c that describes what happens after a deviation, will no longer be a real number but a more complicated object. In particular, the parameter c will be equal to a pair of functions $c = (p, m)$, where $p : [0, 1]^2 \to [0, 1]$ and $m : [0, 1]^2 \to [0, 1]$ are both non-decreasing in each of their two arguments.

2.1. **Norms.** Let $\mathcal{E}$ be a set whose elements are pairs of functions (p, m), where $p : [0, 1]^2 \to [0, 1]$ and $m : [0, 1]^2 \to [0, 1]$ are both non-decreasing in each of their two arguments. We will interpret $\mathcal{E}$ as the set of possible ways in which social norms can be enforced in a given society and call $\mathcal{E}$ the *set of possible norm enforcements* or the *enforcement technology set.*

A *social norm* will be identified with a pair (x, c), where $x \in \mathcal{X}$ is the allocation of surplus specified by the norm and $c \in \mathcal{E}$ describes what happens if agents deviate from the norm.

We want to consider the case where each agent i can attempt to "grab" any fraction $x'_i \in [0, 1]$ of the produced surplus. If the norm $(x, c) \in \mathcal{X} \times \mathcal{E}$ is in place and the agent attempts to grab $x'_i \in [0, 1]$ of the jointly produced surplus, then their attempt is successful with a probability of $1 - p(x'_i - x_i, x_i)$ and is detected by the other player with a probability of $p(x'_i - x_i, x_i)$. If the attempt is successful, then the agent receives x'_i but incurs a monetary cost of $m(x'_i - x_i, x_i)$ which, for instance, could represent the costs of overcoming anxiety after breaking an internalized norm, the disutility from sanctions that deviators face, or costs incurred to avoid such sanctions. What happens if the agent is unsuccessful and their attempt is detected? For now, we will assume that if the other agent detects that their partner wants to behave in a way that is inconsistent with the valid norms, they will cease any cooperation and player i who attempted to deviate from the norm will receive nothing, meaning his or her utility is $u_i(0)$.[4] Note that we said nothing about the payoffs that the other player receives when player i attempts to deviate from the behavior specified by the norm. Thus, our modeling approach, for instance, can handle situations where if player i "grabs" x'_i this leaves the other player with nothing (e.g., in the example discussed in the introduction) and situations where if player i "grabs" $x'_i < 1$ this still leave some positive share of surplus to the other player.

If the norm specifies a division $x \in \mathcal{X}$ and "deviation costs" are captured by $c = (p, m)$, agent i will have no incentive to use the action that gives him x'_i if and only if

$$u_i(x_i) \geq (1 - p(x'_i - x_i, x_i)) \cdot u_i(x'_i - m(x'_i - x_i, x_i)) + p(x'_i - x_i, x_i) \cdot u_i(0).$$

This motivates the following definition.

Definition 1. *An allocation $x \in \mathcal{X}$* can be sustained as part of a norm with $c = (p, m) \in \mathcal{E}$ *if and only if*

$$u_i(x_i) \geq (1 - p(x'_i - x_i, x_i)) \cdot u_i(x'_i - m(x'_i - x_i, x_i)) + p(x'_i - x_i, x_i) \cdot u_i(0) \tag{2}$$

holds for all players $i \in \{1, 2\}$ and all $x'_i \in [0, 1]$. For any allocation $x \in \mathcal{X}$, denote the set of $c \in \mathcal{E}$ for which x can be sustained by $\mathcal{S}_{\mathcal{E}}(x)$.

We restricted attention to functions p and m, which increase in both of their arguments. The fact that p is non-decreasing in the first argument captures the idea that for any given norm it is (weakly) harder to "grab" larger shares of the surplus without being detected. The fact that p increases in the second argument captures the idea that it is (weakly) harder to "grab" a fixed amount from the other player if the other player is getting very little. If m captures the feeling of anxiety after breaking a norm, then the assumption that m is non-decreasing in the first argument captures the idea that larger deviations cause (weakly) more anxiety. Finally, the fact that m is non-decreasing in the second argument captures the idea that a deviation from the norm by a fixed amount may be seen as more justified if one gets a little instead of a lot.

[4]See Section 5.2 for a brief discussion of alternatives.

Remark 1. *Note that the function $\mathcal{S}_{\mathcal{E}}$ introduced in Definition 1 only depends on the preferences of both players and, therefore, not on which utility function is used to represent those preferences. Thus, $\mathcal{S}_{\mathcal{E}}$ will not be affected if positive affine transformations are applied to the utility functions u_1 and u_2. As a result, the same is true for the derived concepts defined in terms of $\mathcal{S}_{\mathcal{E}}$ (e.g., Definition 3 and Definition 4 in the next subsection). Thus, we can – without loss of generality – assume that $u_1(0) = u_2(0) = 0$ and $u_1(1) = u_2(1) = 1$, whenever this is convenient.*

Definition 2. *An enforcement technology set $\mathcal{E}$* is regular *if and only if for any $(p, m) \in \mathcal{E}$ it is the case that the functions p and m are continuous.*

Regular technology sets have the property that for any $c \in \mathcal{E}$ the set of allocations $x \in \mathcal{X}$ which can be sustained as part of a norm using that c is closed in $\mathcal{X}$.

2.2. **An Induced Partial Order on $\mathcal{X}$.** The function $\mathcal{S}_{\mathcal{E}}$ can be used to compare different allocations of surplus in terms of how large the set of enforcement technologies is for which a given allocation can be sustained as part of a norm.

Definition 3. *Fix an enforcement technology set $\mathcal{E}$. An allocation $x \in \mathcal{X}$* is easier to sustain as part of a norm than *$y \in \mathcal{X}$ (we will also use the notation $x \succ_{\mathcal{E}} y$) if and only if $\mathcal{S}_{\mathcal{E}}(y) \subsetneq \mathcal{S}_{\mathcal{E}}(x)$.*

The above definition immediately implies that the binary relation $\succ_{\mathcal{E}}$ on $\mathcal{X}$ is irreflexive (i.e., there is no x with $x \succ_{\mathcal{E}} x$) and transitive (i.e., for $x, y, z \in \mathcal{X}$, $y \succ_{\mathcal{E}} x$ and $z \succ_{\mathcal{E}} y$ implies $z \succ_{\mathcal{E}} x$). Thus, $\succ_{\mathcal{E}}$ is a strict partial order on $\mathcal{X}$.[5]

It will be useful to introduce some language to describe allocations that are the greatest elements with respect to the partial order $\succ_{\mathcal{E}}$.

Definition 4. *Fix an enforcement technology set $\mathcal{E}$. An allocation $x \in \mathcal{X}$* is the easiest to sustain as part of a norm *if and only if $\mathcal{S}_{\mathcal{E}}(y) \subsetneq \mathcal{S}_{\mathcal{E}}(x)$ for all allocations $y \in \mathcal{X}$ satisfying $y \neq x$.*

[5]A binary relation that is irreflexive and transitive is called a strict partial order.

If $x \in \mathcal{X}$ is the easiest allocation to sustain for some enforcement technology set $\mathcal{E}$ then the set of $c \in \mathcal{E}$ for which a player would want to deviate is smaller than for any other allocation $y \neq x$. Thus, an allocation that is the easiest to sustain can be seen as one that is strictly more robust than any other allocation.

2.3. **The Cost-Minimization Problem.** In the case of the example considered in the introduction, the allocation in which each player received half a dollar was not only the allocation which was the easiest to sustain as part of a norm in the sense defined above: It was also the unique allocation that solved a certain cost-minimization problem.

We will say that *a function* $\kappa : \mathcal{E} \to [0, \infty)$ *is non-decreasing* if and only if $\kappa(p, m) \geq \kappa(p', m')$ whenever $p \geq p'$ and $m \geq m'$. Let $\kappa : \mathcal{E} \to [0, \infty)$ be some non-decreasing function and consider the problem

$$\min_{x \in \mathcal{X} : \mathcal{S}_{\mathcal{E}}(x) \neq \emptyset} \inf\{\kappa(c) : c \in \mathcal{S}_{\mathcal{E}}(x)\}. \tag{3}$$

We will call this minimization problem the *cost-minimization problem for the enforcement technology set* $\mathcal{E}$ *and cost function* κ and will be interested in the question whether there exists a unique $x \in \mathcal{X}$ with $\mathcal{S}_{\mathcal{E}}(x) \neq \emptyset$ that minimizes $\inf\{\kappa(c) : c \in \mathcal{S}_{\mathcal{E}}(x)\}$. In other words, we will be interested in the question whether there is an allocation that is cheapest to sustain in that the cost is lowest if we consider the infimum over all enforcement technologies c which can be used to sustain x. The reason why we take $\inf\{\kappa(c) : c \in \mathcal{S}_{\mathcal{E}}(x)\}$ rather than $\min_{c \in \mathcal{S}_{\mathcal{E}}(x)} \kappa(c)$ is that additional assumptions on the set $\mathcal{E}$ are required to guarantee that the latter exists.

Definition 5. *Fix an enforcement technology set* $\mathcal{E}$ *and a non-decreasing cost function* $\kappa : \mathcal{E} \to [0, \infty)$. *We will say that* x *is* the unique solution to the cost-minimization problem *(for that enforcement technology set and cost function) if and only if* x *is the unique solution to the problem (3), in the sense that* $\mathcal{S}_{\mathcal{E}}(x) \neq \emptyset$ *and for any* $y \in \{z \in \mathcal{X} : \mathcal{S}_{\mathcal{E}}(z) \neq \emptyset\}$ *with* $y \neq x$ *it is the case that* $\inf\{\kappa(c) : c \in \mathcal{S}_{\mathcal{E}}(x)\} < \inf\{\kappa(c) : c \in \mathcal{S}_{\mathcal{E}}(y)\}$.

We will later see that there is a relationship between the set of allocations satisfying Definition 4 for some enforcement technology $\mathcal{E}$ and the set of allocations satisfying the above definition for some enforcement technology set $\mathcal{E}$ and some non-decreasing cost function $\kappa : \mathcal{E} \to [0, \infty)$.

3. Examples

In the introduction we saw an example where the allocation in which each player received half a dollar was the easiest to sustain and a unique solution to the cost-minimization problem for some non-decreasing cost function κ.[6] The following examples will yield two other prominent bargaining solutions.

To simplify notation we will assume in this section that $u_i(0) = 0$ for each player i. Under Remark 1 this is without loss of generality.

3.1. Kalai-Smorodinsky Solution.

Let $\mathcal{E}$ be the set of pairs (p, m) such that the function $m : [0,1]^2 \to [0, \infty)$ satisfies $m \equiv 0$ and the function $p : [0,1]^2 \to [0,1]$ satisfies $p \equiv r$ for some $r \in [0,1]$. This means that deviations do not result in any monetary costs. Instead, if an agent tries to "grab" a larger share of the surplus than specified by the norm, there is an exogenously fixed probability r that the bargaining process will permanently end and each player will get their disagreement payoff.

Proposition 1. *For the enforcement technology set $\mathcal{E}$ considered in this subsection, there exists an allocation of surplus that is easier to sustain as a norm than any other allocation of surplus: the Kalai-Smorodinsky solution, meaning the unique allocation $x^{K.S.}$, in which $\frac{u_1(x_1^{K.S.})}{u_1(1)} = \frac{u_2(x_2^{K.S.})}{u_2(1)}$.*

[6] It is straightforward to reproduce this example in our framework where an agent can attempt to grab any amount of surplus. Let $\bar{\mathcal{E}}$ be the set of pairs (p, m) such that $p : [0,1]^2 \to [0,1]$, $m : [0,1]^2 \to [0, \infty)$, and both functions are non-decreasing in each. Define $\mathcal{E}$ as the set of $(p, m) \in \bar{\mathcal{E}}$ such that m is a constant function and p is given by the requirement that $p(\Delta, x_i) = 0$ for $x_i + \Delta \leq 0.60$ and $p(\Delta, x_i) = 1$ for $x_i + \Delta > 0.60$. This means that the agent can attempt to "grab" any amount of surplus but attempts to grab more than \$0.60 will be unsuccessful with probability 1 – and thus never something the agent will want to consider – and attempts to grab \$0.60 or less always successful. The reasoning from the introduction can then be applied unchanged to conclude that the allocation in which each player receives half a dollar is both easiest to sustain and the unique solution to the cost-minimization problem for a natural cost function κ.

Proof. Consider an arbitrary allocation x. Since r does not depend on x', whenever inequality (2) is not satisfied for some player i and $x'_i \in [0,1]$, it will also not hold for that player i and $x'_i = 1$. Thus, $(p,m) \in \mathcal{S}_{\mathcal{E}}(x)$ if and only if

$$u_i(x_i) \geq (1 - p(1, x_i)) \cdot u_i(1)$$

for $i \in \{1,2\}$. This means that

$$\mathcal{S}_{\mathcal{E}}(x) = \{(p,m) \in \mathcal{E} : p(1,0) \in [1 - \min(\frac{u_1(x_1)}{u_1(1)}, \frac{u_2(x_2)}{u_2(1)}), 1]\}.$$

It is easy to see that $\min(\frac{u_1(x_1)}{u_1(1)}, \frac{u_2(x_2)}{u_2(1)})$ achieves its maximum for the Kalai-Smorodinsky solution, that being the unique allocation $x^{K.S.}$ satisfying $\frac{u_1(x_1^{K.S.})}{u_1(1)} = \frac{u_2(x_2^{K.S.})}{u_2(1)}$. □

Note that if it is more costly for a society to implement enforcements with higher probabilities r, the Kalai-Smorodinsky solution will be the unique allocation that is cheapest to sustain. For instance, $x^{K.S.}$ is the unique solution to the cost-minimization problem for $\mathcal{E}$ and the cost function $\kappa : \mathcal{E} \to [0,\infty)$ defined by $\kappa((p,m)) = p(1,0)$ for $(p,m) \in \mathcal{E}$.

3.2. **Nash Bargaining Solution.** In the last subsection we considered the case where whenever a player tried to "grab" a bigger share of the surplus than specified by the norm there is an exogenously fixed positive probability that the bargaining process will permanently end, with each player obtaining their disagreement payoff. In this section, we consider an example where the probability of permanent disagreement considered in the last section depends on how large the deviation from the norm is. Specifically, we will assume that more extreme deviations result in a higher chance of cooperation breaking down permanently.

Let $\mathcal{E}$ be the set of pairs (p,m) such that the function $m : [0,1]^2 \to [0,\infty)$ satisfies $m \equiv 0$ and the function $p : [0,1]^2 \to [0,1]$ is continuous, $p(\Delta, x_i)$ does not depend on x_i, satisfies $p(0, x_i) = 0$, and at points (Δ, x_i) where $p(\Delta, x_i) < 1$, p is differentiable and satisfies $\frac{\partial}{\partial \Delta} p(\Delta, x_i) > 0$ and $\frac{\partial^2}{\partial \Delta^2} p(\Delta, x_i) \geq 0$. The conditions on p capture the idea that: (i) the chance of failure is low for allocations "close to the norm x"; (ii) the chance of failure rises the more excessive the deviation becomes, provided it has not yet reached 1; (iii) the "marginal chance of failure" is larger for more excessive deviations, again, provided the probability of failure has not yet reached 1.

Proposition 2. *For the enforcement technology set $\mathcal{E}$ considered in this subsection, there exists an allocation of surplus that is easier to sustain as a norm than any other allocation of surplus, and that allocation is the symmetric Nash bargaining solution x^{NBS}, namely the unique solution to the problem $\max_{x\in\mathcal{X}}(u_1(x)-u_1(0))\cdot(u_2(x)-u_2(0))$.*

Proof. We claim that, for any allocation $x \in \mathcal{X}$, the set $\mathcal{S}_{\mathcal{E}}(x)$ satisfies

$$\mathcal{S}_{\mathcal{E}}(x) = \{(p,m) \in \mathcal{E} : \frac{\partial}{\partial\Delta}p(0,0)\cdot u_1(x_1) \geq u_1'(x_1) \text{ and } \frac{\partial}{\partial\Delta}p(0,0)\cdot u_2(x_2) \geq u_2'(x_2)\}. \tag{4}$$

To see that (4) holds, consider any allocation $x \in \mathcal{X}$ and any $(p,m) \in \mathcal{E}$. For $i \in \{1,2\}$, define $f_i : [x_i, 1] \to \mathbb{R}$ by $f_i(x_i') = (1 - p(x_i' - x_i, x_i)) \cdot u_i(x_i')$. Now inequality (2) can be rewritten as

$$u_i(x_i) \geq f_i(x_i'). \tag{5}$$

Note that, since $p(0, x_i) = 0$ for any $(p,m) \in \mathcal{E}$ and any $x_i \in [0,1]$, inequality (5) holds with equality if $x_i' = x_i$. Since the functions f_i are concave on the relevant domain,[7] this implies that $f'(x_i) \leq 0$ is a sufficient and necessary condition for inequality (5) to hold for all $x_i' \in (x_i, 1]$, whenever $x_i < 1$. Since $f'(x_i) \leq 0$ is equivalent to $\frac{\partial}{\partial\Delta}p(0,0)\cdot u_i(x_i) \geq u_i'(x_i)$, this proves (4) for the case where $x_1 < 1$ and $x_2 < 1$. However, for the case where $x_1 = 1$ or $x_2 = 1$, equation (4) holds since $\mathcal{S}_{\mathcal{E}}(x) = \emptyset$ and the right-hand side of (4) is also equal to the empty set.[8]

[7] Note that (5) always holds if $p(x_i' - x_i, x_i) = 1$. The fact that $f_i(x_i')$ is concave if we restrict attention to x_i' such that $p(x_i' - x_i, x_i) < 1$ follows from the assumptions on p for $(p,m) \in \mathcal{E}$ and $u_i \geq 0$, $u_i' > 0$, $u_i'' \leq 0$.

[8] Assume $x_i = 1$ for some $i \in \{1,2\}$. $\mathcal{S}_{\mathcal{E}}(x) = \emptyset$ must hold, considering that for any $(p,m) \in \mathcal{E}$ it will be the case that (2) does not hold for player j with $x_j = 0$ and a positive x_j' that are sufficiently close to zero. To see that the right-hand side of (4) is also equal to the empty set, note that if $x_j = 0$, then for any $(p,m) \in \mathcal{E}$, $\frac{\partial}{\partial\Delta}p(0,0)\cdot u_j(x_j) = 0 < u_j'(x_j)$ as $u_j(0) = 0$ and $u_j'(0) > 0$.

Equation (4) implies that an allocation $x \in \mathcal{X}$ is easier to sustain as a norm than an allocation $y \in \mathcal{X}$ if and only if

$$\max(\frac{u_1'(x_1)}{u_1(x_1)}, \frac{u_2'(x_2)}{u_2(x_2)}) < \max(\frac{u_1'(y_1)}{u_1(y_1)}, \frac{u_2'(y_2)}{u_2(y_2)})$$

or, equivalently,

$$\max(\frac{u_1'(x_1)}{u_1(x_1)}, \frac{u_2'(1-x_1)}{u_2(1-x_1)}) < \max(\frac{u_1'(y_1)}{u_1(y_1)}, \frac{u_2'(1-y_1)}{u_2(1-y_1)}).$$

Consider

$$\max(\frac{u_1'(x_1)}{u_1(x_1)}, \frac{u_2'(1-x_1)}{u_2(1-x_1)}) \tag{6}$$

as a function of x_1. Since the functions u_1 and u_2 are concave and increasing, $\frac{u_1'(x_1)}{u_1(x_1)}$ is decreasing in x_1 and $\frac{u_2'(1-x_1)}{u_2(1-x_1)}$ is increasing in x_1. Thus, there is a single allocation for which (6) is minimized and that allocation is the unique solution of the equation

$$\frac{u_1'(x_1)}{u_1(x_1)} = \frac{u_2'(1-x_1)}{u_2(1-x_1)}. \tag{7}$$

However, the unique allocation for which equation (7) holds is the symmetric Nash bargaining solution.[9] □

Given the above analysis, it is straightforward to check that x^{NBS} is the unique solution for the cost-minimization problem for the enforcement technology set $\mathcal{E}$ and the cost function $\kappa : \mathcal{E} \to [0, \infty)$ given by $\kappa((p, m)) = \frac{\partial}{\partial \Delta} p(0, 0)$.

Note that the enforcement technologies considered in this example have a very particular property: The set $\mathcal{S}_{\mathcal{E}}(x)$ depended only on the local properties of the utility functions u_1 and u_2 around x and on the disagreement payoff. This is reminiscent of Nash's (1950) Independence of Irrelevant Alternatives Axiom.

[9]The symmetric Nash bargaining solution is the unique allocation solving $\max_x u_1(x_1) \cdot u_2(1-x_1)$. Note that this problem has an interior solution and that the first order condition $u_1'(x_1) \cdot u_2(1 - x_1) - u_1(x_1) \cdot u_2'(1-x_1) = 0$ is equivalent to equation (7).

4. General Results

4.1. **Set-Valued Solution Concepts.** In Section 3 we saw that for well-established solution concepts like the Nash bargaining solution, the Kalai-Smorodinsky solution, or the equal monetary split, there exist enforcement technology sets $\mathcal{E}$ that yield those bargaining solutions in the sense that the allocation of surplus predicted by the bargaining solution is also the allocation that is the easiest to sustain as part of a social norm and that this allocation solves the cost-minimization problem for some cost function κ.

Definition 6. *Let $\mathcal{X}^{cost\text{-}min}$ be the set of allocations $x \in X$ such that there exists a regular*[10] *enforcement technology set $\mathcal{E}$ and a non-decreasing function $\kappa : \mathcal{E} \to [0, \infty)$ with the property that x is the unique solution to the cost-minimization problem for that enforcement technology set $\mathcal{E}$ and that cost function κ.*

In the language of Myerson (1991), $\mathcal{X}^{\text{cost-min}}$ can be seen as a "lower solution concept" in the sense that for any $x^* \in \mathcal{X}^{\text{cost-min}}$ we can find an environment where x^* is the unique allocation minimizing costs and thus the only predicted allocation if norms are chosen to minimize the costs of sustaining them.

Let us analogously introduce some notation for the set of all allocations that are the easiest to sustain for some regular enforcement technology set $\mathcal{E}$.

Definition 7. *Let $\mathcal{X}^{easiest}$ be the set of allocations $x \in X$ such that there exists a regular enforcement technology set $\mathcal{E}$ with the property that x is the easiest to sustain, meaning $\mathcal{S}_{\mathcal{E}}(y) \subsetneq \mathcal{S}_{\mathcal{E}}(x)$ for all allocations $y \in \mathcal{X}$ satisfying $y \neq x$.*

In the next section we will give an exact characterization of both $\mathcal{X}^{\text{cost-min}}$ and $\mathcal{X}^{\text{easiest}}$.

4.2. **Characterization.** A fundamental question that solution concepts like the Nash bargaining solution or the Kalai-Smorodinsky solution try to address is how different

[10] Regular enforcement technology sets were defined in Definition 2. In Section 4.3 we discuss how the results change if the requirement that $\mathcal{E}$ is regular is dropped in this and the following definition.

attitudes toward risk affect bargaining outcomes. A basic intuition is as follows: If one player is more risk averse than the other player, they will be less aggressive when making demands, allowing the other player to achieve a more favorable outcome.

Our approach will predict that exactly those allocations x are possible bargaining outcomes which are not unbalanced in the sense that, given allocation x, one player will be strictly more willing to make risky demands than the other.

Definition 8. *Let $i \in \{1,2\}$ be a player and j his opponent. Define $\mathcal{D}_i \subset \mathcal{X}$ as the set of allocations x such that for any $q \in (0,1)$ and $\Delta \in (0,1)$,*

$$x_i + \Delta \leq 1 \quad \textit{and} \quad u_i(x_i) < q \cdot u_i(x_i + \Delta) + (1-q) \cdot u_i(0)$$

is implied by

$$x_j + \Delta \leq 1 \quad \textit{and} \quad u_j(x_j) \leq q \cdot u_j(x_j + \Delta) + (1-q) \cdot u_j(0).$$

To understand Definition 8,[11] imagine that a player contemplates accepting allocation x or appealing against x. Specifically, imagine that the player can demand $x'_i = x_i + \Delta$ but the request will be accepted only with some probability q and will result in disagreement with probability $1-q$. The set $\mathcal{D}_i$ is the set of allocations that are unbalanced in the sense that player i would have higher incentives to make such demands than their opponent.[12]

Note that, on an intuitive level, it seems natural for a player to be more willing to appeal an allocation x and demand some x' with $x'_i > x_i$ if x is an allocation that only gives player i a small share of the surplus. This suggests if $x \in \mathcal{D}_i$, then for any allocation y with $y_i < x_i$ it must be that $y \in \mathcal{D}_i$. Lemma 2 in the Appendix formally shows that for each player $i \in \{1,2\}$, there exists a number $\bar{x}_i = \sup_{x \in \mathcal{D}_i} x_i \in \left(0, \frac{1}{2}\right]$, with the property that the set $\mathcal{D}_i$ either satisfies $\mathcal{D}_i = \{x \in \mathcal{X} : x_i < \bar{x}_i\}$ or satisfies $\mathcal{D}_i = \{x \in \mathcal{X} : x_i \leq \bar{x}_i\}$.

[11]See Rubinstein, Safra, & Thomson (1992) for a characterization of the Nash bargaining solution in similar terms.

[12]See Lemma 5 in the Appendix for a different characterization.

Theorem 1. *Let $x \in \mathcal{X}$ be an allocation of surplus. Then, the following three statements are equivalent:*

(i) There exists a regular enforcement technology set $\mathcal{E}$ and a non-decreasing function κ such that x is the unique solution to the cost-minimization problem

$$\min_{x \in \mathcal{X}: \mathcal{S}_{\mathcal{E}}(x) \neq \emptyset} \inf\{\kappa(c) : c \in \mathcal{S}_{\mathcal{E}}(x)\}.$$

(ii) There exists a regular enforcement technology set $\mathcal{E}$ such that x is the easiest allocation of surplus to sustain for the enforcement technology set $\mathcal{E}$.

(iii) For each player i, there is no $y \in \mathcal{D}_i$ with $y_i > x_i$.

In other words, $\mathcal{X}^{\textit{cost-min}} = \mathcal{X}^{\textit{easiest}} = \{x \in \mathcal{X} : x_1 \geq \bar{x}_1$ and $x_2 \geq \bar{x}_2\}$, where $\bar{x}_i = \sup_{x \in \mathcal{D}_i} x_i > 0$ for $i = 1, 2$.[13]

Proof. See the Appendix. □

The proof of Theorem 1 is in the Appendix; here, we just mention some basic ideas used in the proof. Define $\bar{x}_1$ and $\bar{x}_2$ as in the theorem. To prove Theorem 1, it is enough to show that: (1) $\mathcal{X}^{\text{easiest}} \subset \{x \in \mathcal{X} : x_1 \geq \bar{x}_1 \text{ and } x_2 \geq \bar{x}_2\}$, (2) $\{x \in \mathcal{X} : x_1 \geq \bar{x}_1 \text{ and } x_2 \geq \bar{x}_2\} \subset \mathcal{X}^{\text{cost-min}}$, and (3) $\mathcal{X}^{\text{cost-min}} \subset \mathcal{X}^{\text{easiest}}$.

To provide some intuition for the result, let us sketch why (1) holds. To prove (1) it is enough to show that for any allocation $x \in \mathcal{X}$ with the property that $x_i < \bar{x}_i$ holds for some player i, it is the case that $x \notin \mathcal{X}^{\text{easiest}}$. Now, note that if x is an allocation such that $x_i < \bar{x}_i$ then there is an allocation y with the property that $x_i < y_i < \bar{x}_i$. Since $x_i < \bar{x}_i$ and $y_i < \bar{x}_i$, both x and y lie in $\mathcal{D}_i$. But $\mathcal{D}_i$ was the set of allocations z, so that given z, player i has strictly higher incentives to make demands than the other player j. This suggests that, for any enforcement technology set $\mathcal{E}$, $\mathcal{S}_{\mathcal{E}}(x)$ and $\mathcal{S}_{\mathcal{E}}(y)$ will be equal to the set of norm enforcements $c \in \mathcal{E}$ such that player i would not want to impose any alternative allocations. (If player i does not want to impose any alternative allocations, the same is true for the other player j, as they have strictly weaker incentives to make demands.)[14] However, if $\mathcal{S}_{\mathcal{E}}(x)$ and $\mathcal{S}_{\mathcal{E}}(y)$ are

[13] The sets $\mathcal{D}_i$ were defined in Definition 8.

[14] See Lemma 3 in the Appendix for a formal statement.

both determined only by player i's incentives, then we expect that $\mathcal{S}_{\mathcal{E}}(x) \subset \mathcal{S}_{\mathcal{E}}(y)$ will hold for any $\mathcal{E}$. (Indeed, as $x_i < y_i$ holds, we expect player i will be more satisfied under y than under x and will have weaker incentives to make demands under y than under x.) This, however, implies $x \notin \mathcal{X}^{easiest}$. To prove statement (2) for each $x \in \{x \in \mathcal{X} : x_1 \geq \bar{x}_1 \text{ and } x_2 \geq \bar{x}_2\}$ an enforcement technology set $\mathcal{E}$ and a cost function κ are constructed so that x is the unique solution to the cost-minimization problem for that enforcement technology set and cost function. The proof of statement (3) is relatively straightforward.

The set $\mathcal{X}^{\text{cost-min}} = \mathcal{X}^{\text{easiest}}$ characterized in Theorem 1 in general depends on the risk preferences of the two players. For instance, it is straightforward to show that, for the case where both players have the same preferences over lotteries, $\mathcal{X}^{\text{cost-min}} = \mathcal{X}^{\text{easiest}} = \{(\frac{1}{2}, \frac{1}{2})\}$ holds. From Theorem 1 and the examples considered in Section 3 we know that, in general, the set $\mathcal{X}^{\text{cost-min}} = \mathcal{X}^{\text{easiest}}$ is convex and contains the Nash bargaining solution, the Kalai-Smorodinsky solution, and the equal division $(\frac{1}{2}, \frac{1}{2})$. The reader might wonder how much larger $\mathcal{X}^{\text{cost-min}} = \mathcal{X}^{\text{easiest}}$ is compared to the convex hull of those three allocations. The following proposition addresses this question.

Proposition 3. *For $i = 1, 2$, let $\bar{y}_i \in [0, 1]$ be the unique solution of*

$$\frac{\frac{u_i(2\bar{y}_i)-u_i(\bar{y}_i)}{\bar{y}_i}}{u_i(\bar{y}_i) - u_i(0)} = \frac{u_j'(1-\bar{y}_i)}{u_j(1-\bar{y}_i) - u_j(0)},$$

where j stands for the other player. Define $\mathcal{Y}^* \subset \mathcal{X}$ by

$$\mathcal{Y}^* = \{x \in \mathcal{X} : x_1 \geq \min(\bar{y}_1, \frac{1}{2}) \text{ and } x_2 \geq \min(\bar{y}_2, \frac{1}{2}).\}$$

Then, $\mathcal{X}^{cost\text{-}min} = \mathcal{X}^{easiest} \subset \mathcal{Y}^*$.

Proof. See the Appendix. □

Consider the equation defining $\bar{y}_i$. Note that if we replace $\frac{u_i(2\bar{y}_i)-u_i(\bar{y}_i)}{\bar{y}_i}$ with $u_i'(\bar{y}_i) \geq \frac{u_i(2\bar{y}_i)-u_i(\bar{y}_i)}{\bar{y}_i}$ we obtain an equation which characterizes the payoff of player i under the

Nash bargaining solution.[15] Thus, with $\mathcal{Y}^*$ we have an outer solution concept whose extreme points can be directly related to the Nash bargaining solution.

One can show that for the case where one of the players is risk neutral the set $\mathcal{Y}^*$ defined in Proposition 3 actually coincides with $\mathcal{X}^{\text{cost-min}} = \mathcal{X}^{\text{easiest}}$. This implies that if player i is risk neutral then $\mathcal{X}^{\text{cost-min}} = \mathcal{X}^{\text{easiest}} = \{x \in \mathcal{X} : x_j \in [\bar{y}_i, \frac{1}{2}]\}$ where $\bar{y}_i$ is defined as in Proposition 3 and j stands for the other player.

4.3. **Robustness of the Characterization.** In Definition 6 and Definition 7 we required that the enforcement technologies sets are regular.

Theorem 2. *Statements (i), (ii), and (iii) in Theorem 1 are still equivalent for any allocation $x \in \mathcal{X}$ if the word "regular" is dropped in (i) and (ii).*

Proof. See the Appendix. □

Note that neither of the two theorems immediately implies the other. The less restrictive we are in terms of the permitted enforcement technologies, the larger the sets of allocations satisfying (i) and (ii) potentially get. Thus, in Theorem 1 the statement that (iii) implies (i) and (ii) does not follow directly from the fact that (iii) implies (i) and (ii) in the context of Theorem 2. Similarly, the fact that (i) and (ii) imply (iii) in Theorem 2 does not follow from the fact that (i) and (ii) imply (iii) in the context of Theorem 1.

Theorem 2 could be proven independently, following the same lines as the proof of Theorem 1 in the appendix, where the proof of statement (2) could be simplified by using enforcement technologies that are not continuous. The fact that the proof of Theorem 1 is slightly more involved is one of several reasons why Theorem 1 is presented as the main result, with Theorem 2 serving as a robustness check.

[15]The Nash bargaining solution is the allocation x that maximizes $(u_1(x_1) - u_1(0)) \cdot (u_2(x_2) - u_2(0))$. It therefore satisfies the first-order condition $\frac{u_1'(x_1)}{u_1(x_1)-u_1(0)} = \frac{u_2'(x_2)}{u_2(x_2)-u_2(0))}$. Replacing x_j with $1 - x_i$ we obtain the equation.

4.4. **Comparative Statics.** According to Theorem 1, $\mathcal{X}^{\text{cost-min}} = \mathcal{X}^{\text{easiest}} = \{x \in \mathcal{X} : x_1 \geq \bar{x}_1 \text{ and } x_2 \geq \bar{x}_2\}$, where $\bar{x}_i = \sup_{x \in \mathcal{D}_i} x_i$ for $i = 1, 2$. The definition of the sets $\mathcal{D}_i$ implies some immediate comparative statics results.

Consider, for instance, how the set $\mathcal{X}^{\text{cost-min}} = \mathcal{X}^{\text{easiest}}$ changes if player 1 became more risk averse in that their preference over risk is no longer given by u_1 but $\widehat{u}_1 = v \circ u_1$ instead, where v is an increasing, strictly concave function. The definition of the sets $\mathcal{D}_i$ together with Jensen's inequality immediately implies that $\bar{x}_1$ would weakly decrease and $\bar{x}_2$ would weakly increase. Thus, $\mathcal{X}^{\text{cost-min}} = \mathcal{X}^{\text{easiest}}$ would "shift" in player 2's favor.

4.5. **Dominance.** Let $\mathcal{E}$ be an enforcement technology set. So far, we have focused on the case where the cost-minimization problem (3) has a unique solution. Consider the case where for some regular enforcement technology set $\mathcal{E}$ and some cost function κ the minimization problem

$$\min_{x \in \mathcal{X} : \mathcal{S}_{\mathcal{E}}(x) \neq \emptyset} \inf\{\kappa(c) : c \in \mathcal{S}_{\mathcal{E}}(x)\} \tag{8}$$

does not have a unique solution. Can we argue that among the allocations x that minimise costs, some are less attractive than others?

Definition 9. *An allocation $y \in \mathcal{X}$* dominates *an allocation $x \in \mathcal{X}$ if and only if for any regular enforcement technology set $\mathcal{E}$ it is the case that $\mathcal{S}_{\mathcal{E}}(x) \subset \mathcal{S}_{\mathcal{E}}(y)$ and there exists a regular enforcement technology set $\mathcal{E}'$ such that $\mathcal{S}_{\mathcal{E}'}(x) \subsetneq \mathcal{S}_{\mathcal{E}'}(y)$. An allocation is* undominated *if there is no allocation $y \in \mathcal{X}$ that dominates it.*

If an allocation y dominates an allocation x, then for any regular enforcement technology set $\mathcal{E}$ and cost function κ, whenever x is a solution to the minimization problem (8) so is y. Moreover, for any $c \in \mathcal{S}_{\mathcal{E}'}(x)$, implementing (y, c) instead of (x, c) would have the advantage that it achieves the same cost but is more robust. For instance, if agents are occasionally confused about the enforcement technology used, there would be an advantage in using y, because there are enforcement technology sets $\mathcal{E}'$ for which $\mathcal{S}_{\mathcal{E}'}(x) \subsetneq \mathcal{S}_{\mathcal{E}'}(y)$ but there is no downside as $\mathcal{S}_{\mathcal{E}'}(x) \subset \mathcal{S}_{\mathcal{E}'}(y)$ always holds.

Theorem 3. *An allocation $x \in \mathcal{X}$ is undominated if and only if x lies in $\{x \in \mathcal{X} : x_1 \geq \bar{x}_1$ and $x_2 \geq \bar{x}_2\}$, where $\bar{x}_i = \sup_{x \in \mathcal{D}_i} x_i > 0$ for $i = 1, 2$, and $\mathcal{D}_i$ are defined as in Definition 8.*

Proof. See the Appendix. □

Let $\mathcal{X}^{\text{undom}}$ be the set of allocations that are undominated. For Theorem 1 to imply Theorem 3 one just needs to show that $\mathcal{X}^{undom} = \mathcal{X}^{\text{easiest}}$.

Since an allocation x lies in $\mathcal{X}^{\text{easiest}}$ if and only if it is easiest to implement for some regular technology set $\mathcal{E}$, the definition of $\mathcal{X}^{\text{undom}}$ immediately implies that $\mathcal{X}^{\text{easiest}} \subset \mathcal{X}^{\text{undom}}$. However, $\mathcal{X}^{\text{undom}} \subset \mathcal{X}^{\text{easiest}}$ does not follow immediately from the definitions of the two sets and, for instance, will not hold in general if the definition of a regular technology set (Definition 2 in Subsection 2.1) is made sufficiently more restrictive. Why? The the definition of $\mathcal{X}^{\text{undom}}$ guarantees that if $x \in \mathcal{X}^{\text{undom}}$ holds then there is no y such that (i) for any regular $\mathcal{E}$ it is the case that $\mathcal{S}_{\mathcal{E}}(x) \subset \mathcal{S}_{\mathcal{E}}(y)$ and (ii) there exists a regular $\mathcal{E}'$ such that $\mathcal{S}_{\mathcal{E}'}(x) \subsetneq \mathcal{S}_{\mathcal{E}'}(y)$. However, it could potentially be that for some $x \in \mathcal{X}^{\text{undom}}$ there is a $y \neq x$ such that for any regular $\mathcal{E}$ it is the case that $\mathcal{S}_{\mathcal{E}}(x) = \mathcal{S}_{\mathcal{E}}(y)$. If this was the case, then such a $x \in \mathcal{X}^{\text{undom}}$ would not be an element of $x \in \mathcal{X}^{\text{easiest}}$ as x can never be easiest to implement if $\mathcal{S}_{\mathcal{E}}(x) = \mathcal{S}_{\mathcal{E}}(y)$ always holds. The proof in the appendix shows that $\mathcal{X}^{\text{undom}} - \mathcal{X}^{\text{easiest}}$ is empty by showing that if an allocation x is not in $\mathcal{X}^{\text{easiest}}$ then there exists an allocation y that dominates x.

5. Discussion and Extensions

5.1. **Asymmetric Enforcement Technologies.** In our analysis we assumed that players are symmetric in all ways except in their utility functions u_i. This is natural if we want to compare our results with symmetric bargaining solutions like the symmetric Nash bargaining or the Kalai-Smorodinsky solution.

Of course, there are also situations where it is natural to consider asymmetric enforcement technologies. To show how the approach proposed in this paper can be applied to study these situations analogously, consider again the example discussed in the introduction where two players can produce a good worth \$1 without costs but have opportunities to steal the unfinished good before production is complete, and can sell it for \$0.60 if they do so. Imagine now that because the players have different roles in the production process or due to different skillsets, it is easier for player 1 to steal the unfinished good than player 2. Specifically, assume that while player 1 can steal the unfinished good without incurring any costs, player 2 needs to incur a cost of \$0.05 to steal it. Thus, if the norm specifies a division of surplus $x = (x_1, x_2)$, and stealing the good will result in a "punishment" that corresponds to m dollars, player 1 would prefer to steal the good if and only if $x_1 < 0.60 - m$, and player 2 would prefer to steal the good if and only if $x_2 < 0.60 - 0.05 - m$. Thus, no agent will have an incentive to steal the good if and only if

$$m \geq \max(0.60 - x_1, 0.60 - 0.05 - x_2) = 0.60 - \min(x_1, x_2 + 0.5).$$

Analogously as in the introduction, for any allocation x, let $\mathcal{S}(x)$ be the set of non-negative numbers m such that the allocation x is sustainable as a norm in the sense that no player would have an incentive to steal the good, i.e. the above inequality holds for $i = 1$ and $i = 2$. Then,

$$\mathcal{S}(x) = [0.60 - \min(x_1, x_2 + 0.5), \infty).$$

Clearly, the allocation in which player 1 receives \$0.525 and player 2 receives \$0.475 is easiest to implement in the sense that for any allocation y with $y \neq x$, $\mathcal{S}(y) \subsetneq \mathcal{S}(x)$ holds. As in the introduction, if the costs a society incurs are increasing in m this would also be the allocation that would be the cheapest to sustain as part of a norm.

Therefore, in the above example, our model predicts that more skilled individuals (or for other reasons for whom it is easier to "steal" a fraction of the surplus) will receive a higher share of the surplus under the cost-minimizing norm. Of course, we expect this to hold much more generally.

It is also not difficult to use the proposed approach to generate some well-known asymmetric bargaining solutions like the *asymmetric Nash bargaining solution.* To see this, define $\mathcal{E}$ as in Section 3.2 but imagine that if $(p, m) \in \mathcal{E}$ is used, the norm specifies an allocation x, and if player 1 tries to "grab" x_1' his attempt will be unsuccessful with probability

$$\gamma_1 \cdot p(x_1' - x_1, x_1)$$

while if player 2 tries to "grab" x_2' his attempt will be unsuccessful with probability

$$\gamma_2 \cdot p(x_2' - x_2, x_2)$$

where $\gamma_1, \gamma_2 \in (0, 1)$ are constants. Generalizing the ideas from Section 2.1 we can then say that an allocation $x \in \mathcal{X}$ can be sustained as part of a social norm if and only if

$$(9)\ \ u_1(x_1) \geq (1-\gamma_1 \cdot p(x_1'-x_1, x_1)) \cdot u_1(x_1'-m(x_1'-x_1, x_1)) + \gamma_1 \cdot p(x_1'-x_1, x_1) \cdot u_1(0).$$

holds for all $x_1' \in [0, 1]$ and

$$(10)\ \ u_2(x_2) \geq (1-\gamma_2 \cdot p(x_2'-x_2, x_2)) \cdot u_2(x_2'-m(x_2'-x_2, x_2)) + \gamma_2 \cdot p(x_2'-x_2, x_2) \cdot u_2(0).$$

holds for all $x_2' \in [0, 1]$. Using exactly the same arguments as in Section 3.2, it is straightforward to verify that if we define $\mathcal{S}_{\mathcal{E}}^{\gamma_1,\gamma_2}(x)$ as being the set of all allocations $x \in \mathcal{X}$ such that (9) and (10) are satisfied, there will be a unique allocation x^* that is the easiest to sustain – in the sense that $\mathcal{S}_{\mathcal{E}}^{\gamma_1,\gamma_2}(x) \subsetneq \mathcal{S}_{\mathcal{E}}^{\gamma_1,\gamma_2}(x^*)$ for any allocation $x \neq x$ – and that allocation is the asymmetric Nash bargaining solution for weights $\alpha = \frac{\gamma_2}{\gamma_1+\gamma_2}$ and $\beta = \frac{\gamma_1}{\gamma_1+\gamma_2}$, meaning the unique allocation solving:

$$\max_{x \in X} u_1(x_1)^{\frac{\gamma_2}{\gamma_1+\gamma_2}} \cdot u_2(x_2)^{\frac{\gamma_1}{\gamma_1+\gamma_2}}.$$

The above two examples suggest that, conceptually, it is easy to generalize the proposed approach to situations in which enforcement technologies are asymmetric.

5.2. **Other Symmetric Enforcement Technologies.** The fundamental goal of this paper is to describe a novel way in which one can think about outcomes in certain bargaining situations. There is no question that, for specific applications, it might be worthwhile to consider enforcement technologies that are different from those considered here.

For instance, in Section 2.1, we assumed that if an attempt to "grab" a share x_i' of the surplus was unsuccessful, then the result would be disagreement. Alternatively, one could imagine that in such a situation not only would cooperation be permanently ended but the deviator would also face some sanction such as, for instance, incurring a cost of $K > 0$. Analogously to Section 2.1, we could then say that an allocation $x \in \mathcal{X}$ can be sustained as part of a norm if and only if

$$u_i(x_i') \geq (1 - p(x_i' - x_i, x_i)) \cdot u_i(x_i' - m(x_i' - x_i, x_i)) + p(x_i' - x_i, x_i) \cdot u_i(-K).$$

holds for all players i and all $x_i' \in [0,1]$. The analysis could then be carried out along the same lines as was done in Section 4 and Theorem 1 could be derived for similarly defined sets $\mathcal{D}_i$ where, however, in Definition 8, the payoff $u(0)$ would be replaced with the payoff $u(-K)$. Of course, the results would then be a bit harder to compare with standard solution concepts as the derived solution concept would depend on $u_i(-K)$, the disutility from a sanction that is not directly related to the "disagreement payoff" which players' receive if they decide not to cooperate.

The technical reason why this extension would be relatively straightforward is that a fundamental monotonicity property used in our analysis would still hold. The relevant property is that, for any enforcement technology c, a player i has weaker incentives to deviate if the norm prescribes to them a higher rather than lower share of the surplus. (For the framework considered in this paper, this property is formally proven in Lemma 1 in the Appendix.) For some other modifications that one could consider – for instance, if the utility of a player i who attempts to "grab" x_i' and fails is given by $u_i(-m(x_i' - x_i, x_i))$ – additional assumptions on the utility functions u_i are needed to guarantee that the aforementioned monotonicity property holds.

6. Conclusion

In the context of two-player bargaining under symmetric information,[16] this paper studied bargaining outcomes in societies where cooperation is governed by social norms. It proposed a new approach in which bargaining outcomes are analyzed based on how costly they are to sustain as part of a social norm. It showed that well-known bargaining solutions like the Nash bargaining solution or the Kalai-Smorodinsky solution can be understood as unique solutions to the problem of choosing an allocation that is the cheapest to sustain as a norm for some enforcement technologies and cost functions. Moreover, all allocations with the property of being a unique solution to some such cost-minimization problem were characterized.

The questions of whether and when norms would form was not addressed in this paper. This is an important question. However, if an important role of social norms is indeed to "compensate for market failures," then addressing this question will most likely require assumptions regarding the social costs of those "market failures." In contrast, our analysis required no such assumptions – since we only analyzed the incentives of agents to deviate from a norm it was irrelevant to know how the payoffs of other agents are affected by deviations.

Mathematical Appendix

6.1. **Proof of Theorem 1.** Define $\bar{x}_1$ and $\bar{x}_2$ as in the statement of the theorem. As noted in the main part of the paper, to prove the equivalence of (i), (ii), and (iii), it is enough to show:

(1) $\mathcal{X}^{\text{easiest}} \subset \{x \in \mathcal{X} : x_1 \geq \bar{x}_1 \text{ and } x_2 \geq \bar{x}_2\}$,

(2) $\{x \in \mathcal{X} : x_1 \geq \bar{x}_1 \text{ and } x_2 \geq \bar{x}_2\} \subset \mathcal{X}^{\text{cost-min}}$, and

(3) $\mathcal{X}^{\text{cost-min}} \subset \mathcal{X}^{\text{easiest}}$.

Under Remark 1 it is enough to prove (1), (2), and (3) for the case where $u_i(0) = 0$ for $i \in \{1, 2\}$. Assume $u_i(0) = 0$ for $i \in \{1, 2\}$.

[16] In a companion paper bargaining between more players is considered.

For any player $i \in \{1,2\}$ and allocation $x \in \mathcal{X}$, let $\mathcal{S}^i_{\mathcal{E}}(x)$ be the set of $(p,m) \in \mathcal{E}$ with the property that

$$u_i(x_i) \geq (1 - p(x'_i - x_i, x_i)) \cdot u_i(x'_i - m(x'_i - x_i, x_i)) \tag{11}$$

holds for all allocations $x' \in \mathcal{X}$ with $x'_i > x_i$. On an intuitive level, $\mathcal{S}^i_{\mathcal{E}}(x)$ is exactly equal to the set of $c \in \mathcal{E}$ for which player i would not want to impose some alternative outcome $x' \in \mathcal{X}$. The definition of $\mathcal{S}_{\mathcal{E}}(x)$ immediately implies that $\mathcal{S}_{\mathcal{E}}(x) = \mathcal{S}^1_{\mathcal{E}}(x) \cap \mathcal{S}^2_{\mathcal{E}}(x)$.

The following lemma formalizes the intuition that a player who receives more will have smaller incentives to impose an alternative allocation.

Lemma 1. *Let $i \in \{1,2\}$. If $x, y \in \mathcal{X}$ satisfy $x_i < y_i$ then $\mathcal{S}^i_{\mathcal{E}}(x) \subset \mathcal{S}^i_{\mathcal{E}}(y)$.*

Proof of Lemma 1. Let $x, y \in \mathcal{X}$ be such that $x_i < y_i$. We need to show that $\mathcal{S}^i_{\mathcal{E}}(x) \subset \mathcal{S}^i_{\mathcal{E}}(y)$. Assume this is not the case, meaning there exists a $c = (m,p) \in \mathcal{E}$ with the property that $c \in \mathcal{S}^i_{\mathcal{E}}(x)$ and $c \notin \mathcal{S}^i_{\mathcal{E}}(y)$. As $c \notin \mathcal{S}_i(y)$, there exists a $y' \in \mathcal{X}$ with $y'_i > y_i$ such that

$$u_i(y_i) < (1 - p(y'_i - y_i, y_i)) \cdot u_i(y'_i - m(y'_i - y_i, y_i)) \tag{12}$$

or, equivalently,[17]

$$u_i(y_i) \cdot \frac{1}{1 - p(y'_i - y_i, y_i)} < u_i(y'_i - m(y'_i - y_i, y_i)).$$

Subtracting $u_i(y_i)$ from both sides, we see that the above inequality (and, therefore, also inequality (12)) is equivalent to:

$$u_i(y_i) \cdot (\frac{1}{1 - p(y'_i - y_i, y_i)} - 1) < u_i(y'_i - m(y'_i - y_i, y_i)) - u_i(y_i). \tag{13}$$

Let $x' \in \mathcal{X}$ be given by $x'_i - x_i = y'_i - y_i$.[18] Note that:

(a) $u_i(x_i) < u_i(y_i)$ as u_i is increasing and $x_i < y_i$.

[17]Note that, since $u_i(y_i) \geq 0$, the last inequality implies $1 - p(y'_i - y_i, y_i) > 0$.
[18]Such x' exists since $x_i < y_i$ holds and $y'_i \leq 1$.

(b) $(\frac{1}{1-p(x_i'-x_i,x_i)}-1) \leq (\frac{1}{1-p(y_i'-y_i,y_i)}-1)$ as $p(x_i'-x_i,x_i) = p(y_i'-y_i,x_i) \leq p(y_i'-y_i,y_i)$.[19]

(c) $u_i(x_i' - m(x_i' - x_i, x_i)) - u_i(x_i) \geq u_i(y_i' - m(y_i' - y_i, y_i)) - u_i(y_i)$. Indeed, the fact that u_i is concave together with $x_i' - x_i = y_i' - y_i$ implies $u_i(x_i' - m(x_i' - x_i, x_i)) - u_i(x_i) \geq u_i(y_i' - m(y_i' - y_i, x_i)) - u_i(y_i)$. But $u_i(y_i' - m(y_i' - y_i, x_i)) - u_i(y_i) \geq u_i(y_i' - m(y_i' - y_i, y_i)) - u_i(y_i)$ holds as $y_i > x_i$, m is non-decreasing in its second argument, and u_i is increasing.

Inequality (13) together with (a)-(c) imply that

$$u_i(x_i) \cdot (\frac{1}{1 - p(x_i' - x_i, x_i)} - 1) < u_i(x_i' - m(x_i' - x_i, x_i)) - u_i(x_i)$$

or, equivalently

$$u_i(x_i) < (1 - p(x_i' - x_i), x_i) \cdot u_i(x_i' - m(x_i' - x_i, x_i))$$

which contradicts $c \in \mathcal{S}_i(x)$. □

The next lemma characterizes the sets $\mathcal{D}_i$ for $i = 1, 2$.

Lemma 2. *Let $i \in \{1, 2\}$. There exists a number $\bar{x}_i \in (0, \frac{1}{2}]$ with the property that either*

$$\mathcal{D}_i = \{x \in \mathcal{X} : x_i < \bar{x}_i\}$$

or

$$\mathcal{D}_i = \{x \in \mathcal{X} : x_i \leq \bar{x}_i\}.$$

In particular, the set $\mathcal{D}_i$ is non-empty.

Proof of Lemma 2. Let $i \in \{1, 2\}$. We will organize the argument in several steps.

Step 1: Note that the definition of the set $\mathcal{D}_i$ immediately implies that $\mathcal{D}_i$ contains the allocation x with $x_i = 0$. Since $\mathcal{D}_i \subset \mathcal{X}$ is non-empty, we can define $\bar{x}_i$ by $\bar{x}_i = \sup_{x \in \mathcal{D}_i} x_i$.

Step 2: Note that if $x \in \mathcal{D}_i$, then also $x' \in \mathcal{D}_i$ for any $x' \in \mathcal{X}$ with $x_i' < x_i$. To see that this is the case, consider the definition of the set $\mathcal{D}_i$ and note that, for $q \in (0, 1)$

[19] $p(y_i' - y_i, x_i) \leq p(y_i' - y_i, y_i)$ follows from the fact that p is non-decreasing in its arguments and $x_i < y_i$.

and $\Delta \in [0,1]$,

$$u_i(x_i) < q \cdot u_i(x_i + \Delta) + (1-q) \cdot u_i(0)$$

is equivalent to

$$\frac{u_i(x_i) - u_i(0)}{u_i(x_i + \Delta) - u_i(x_i)} < \frac{q}{1-q}$$

and, similarly,

$$u_j(x_j) \leq q \cdot u_j(x_j + \Delta) + (1-q) \cdot u_j(0)$$

is equivalent to

$$\frac{u_j(x_j) - u_j(0)}{u_j(x_j + \Delta) - u_j(x_j)} \leq \frac{q}{1-q}.$$

Our claim now follows immediately from the observation that, since the utility functions of both players are increasing and concave, for $k \in \{1,2\}$, $u_k(x_k + \Delta) - u_k(x_k)$ is non-increasing in x_k and $u_k(x_k) - u_k(0)$ is increasing in x_k.

Steps 1 and 2 together imply that for $\bar{x}_i = \sup_{x \in \mathcal{D}_i} x_i$ either $\mathcal{D}_i = \{x \in \mathcal{X} : x_i < \bar{x}_i\}$ or $\mathcal{D}_i = \{x \in \mathcal{X} : x_i \leq \bar{x}_i\}$ and $\bar{x}_i \geq 0$. All that remains to be shown is that $0 < \bar{x}_i \leq \frac{1}{2}$.

Step 3: To show that $\bar{x}_i \leq \frac{1}{2}$, assume that is not the case and let $x \in \mathcal{D}_i$ be an allocation with $x_i > \frac{1}{2}$. Let $j \in \{1,2\}$ with $j \neq i$. Note that for $\Delta = x_i$ and $q = u_j(x_j)/u_j(x_j + \Delta)$ we have $x_j + \Delta = 1 \leq 1$ and $u_j(x_j) = q \cdot u_j(x_j + \Delta)$. Since $x \in \mathcal{D}_i$, this implies that $x_i + \Delta = 2 \cdot x_i \leq 1$. $2 \cdot x_i \leq 1$, however, contradicts $x_i > \frac{1}{2}$.

Step 4: To show that $\bar{x}_i > 0$, note that for any allocation x such that $0 < x_i < \frac{1}{2}$ and

$$\frac{u_i(2 \cdot x_i) - u_i(0)}{u_i'(1)} < \frac{u_j(\frac{1}{2}) - u_j(0)}{u_i'(0)} \tag{14}$$

it will be the case that $x \in \mathcal{D}_i$.

To see this, assume x is an allocation satisfying the above conditions and that

$$x_j + \Delta \leq 1 \quad \text{and} \quad u_j(x_j) \leq q \cdot u_j(x_j + \Delta) + (1-q) \cdot u_j(0) \tag{15}$$

holds for some $q \in (0,1)$ and $\Delta \in (0,1)$. As $x_j = 1 - x_i$ together with the left inequality in (15) and $x_i < \frac{1}{2}$ implies

$$\Delta \leq x_i \quad \text{and} \quad x_i + \Delta \leq 1. \tag{16}$$

Now, note that the right inequality in (15) is equivalent to

$$1 - q \leq \frac{u_j(x_j + \Delta) - u_j(x_j)}{u_j(x_j + \Delta) - u_j(0)}. \tag{17}$$

Using the concavity of u_1 and u_2, $x_i < \frac{1}{2}$, (14), and (16) we obtain

$$\begin{aligned} \frac{u_j(x_j + \Delta) - u_j(x_j)}{u_j(x_j + \Delta) - u_j(0)} &\leq \frac{\Delta \cdot u_j'(0)}{u_j(\frac{1}{2}) - u_j(0)} < \\ < \frac{\Delta \cdot u_i'(1)}{u_i(2 \cdot x_i) - u_i(0)} &\leq \frac{u_i(x_i + \Delta) - u_i(x_i)}{u_i(x_i + \Delta) - u_i(0)} \end{aligned} \tag{18}$$

Combining (16), (17), and (18) we obtain

$$x_i + \Delta \leq 1 \quad \text{and} \quad u_i(x_i) < q \cdot u_i(x_i + \Delta) + (1 - q) \cdot u_i(0) \tag{19}$$

This proves that for any allocation x such that $0 < x_i < \frac{1}{2}$ and (14) holds it will be the case that $x \in \mathcal{D}_i$. $\square$

The next lemma relates the sets $\mathcal{D}_i$ to $\mathcal{S}^1_{\mathcal{E}}$ and $\mathcal{S}^2_{\mathcal{E}}$. This lemma is the key observation in the proof of statement (1).

Lemma 3. *Fix an enforcement technology set $\mathcal{E}$. Let $i, j \in \{1,2\}$ with $i \neq j$. For any $x \in \mathcal{D}_i$, it is the case that*

$$\mathcal{S}^i_{\mathcal{E}}(x) \subset \mathcal{S}^j_{\mathcal{E}}(x).$$

Proof of Lemma 3. We will prove the statement in the lemma for the case where $i = 1$ and $j = 2$. The argument for the case where $j = 1$ and $i = 2$ is analogous. To show that for any $x \in \mathcal{D}_1$ it is the case that

$$\mathcal{S}^1_{\mathcal{E}}(x) \subset \mathcal{S}^2_{\mathcal{E}}(x),$$

it is enough to show that $(p,m) \notin \mathcal{S}^2_{\mathcal{E}}(x)$ implies $(p,m) \notin \mathcal{S}^1_{\mathcal{E}}(x)$. Assume, therefore, $(p,m) \notin \mathcal{S}^2_{\mathcal{E}}(x)$.

Since $(p,m) \notin \mathcal{S}^2_{\mathcal{E}}(x)$ there must exist a $x_2' \in (x_2, 1]$ such that

$$u_2(x_2) < (1 - p(x_2' - x_2, x_2)) \cdot u_2(x_2' - m(x_2' - x_2, x_2)). \tag{20}$$

Note that (20) implies that $x_2' - m(x_2' - x_2, x_2) > x_2$. Set $\Delta = x_2' - m(x_2' - x_2, x_2) - x_2$ and $q = 1 - p(x_2' - x_2, x_2)$. Note that $x_2' - m(x_2' - x_2, x_2) \leq 1$ implies $x_2 + \Delta \leq 1$. We can now rewrite (20) as

$$u_2(x_2) < q \cdot u_2(x_2 + \Delta).$$

Since $x \in \mathcal{D}_1$, the last inequality together with $x_2 + \Delta \leq 1$ implies that $x_1 + \Delta \leq 1$ and

$$u_1(x_1) < q \cdot u_1(x_1 + \Delta). \tag{21}$$

Let x'' be the allocation characterized by $x_1'' - x_1 = x_2' - x_2$. Note that such an allocation does indeed exist as $x_1 \leq x_2$ follows from Lemma 2 given that $x \in \mathcal{D}_1$. Note that $x_1'' - x_1 = x_2' - x_2$ together with $x_1 \leq x_2$ implies that $\Delta = x_2' - m(x_2' - x_2, x_2) - x_2 = x_1'' - m(x_1'' - x_1, x_2) - x_1 \leq x_1'' - m(x_1'' - x_1, x_1) - x_1$ and $q = 1 - p(x_2' - x_2, x_2) = 1 - p(x_1'' - x_1, x_2) \leq 1 - p(x_1'' - x_1, x_1)$. Thus, inequality (21) implies

$$u_1(x_1) < (1 - p(x_1'' - x_1, x_1)) \cdot u_1(x_1'' - m(x_1'' - x_1, x_1)).$$

Since $x_1'' - x_1 = x_2' - x_2 > 0$ and $x_1'' = x_1 + x_2' - x_2 \leq x_2' \leq 1$, this proves that $(p,m) \notin \mathcal{S}^1_{\mathcal{E}}(x)$, which is what we wanted to show.

$\square$

Lemma 4. *Let $x, y \in \mathcal{D}_i$ with $x_i < y_i$, where $i \in \{1,2\}$. Then, $\mathcal{S}_{\mathcal{E}}(x) \subset \mathcal{S}_{\mathcal{E}}(y)$ holds for any enforcement technology set $\mathcal{E}$.*

Proof of Lemma 4. We will prove the lemma for the case where $i = 1$. The argument for the case $i = 2$ is analogous.

According to Lemma 3,

$$\mathcal{S}^1_{\mathcal{E}}(y) \subset \mathcal{S}^2_{\mathcal{E}}(y)$$

and

$$\mathcal{S}^1_{\mathcal{E}}(x) \subset \mathcal{S}^2_{\mathcal{E}}(x).$$

Note that according to Lemma 1, $\mathcal{S}^1_{\mathcal{E}}(x) \subset \mathcal{S}^1_{\mathcal{E}}(y)$ and $\mathcal{S}^2_{\mathcal{E}}(y) \subset \mathcal{S}^2_{\mathcal{E}}(x)$. Thus,

$$\mathcal{S}_{\mathcal{E}}(x) = \mathcal{S}^1_{\mathcal{E}}(x) \cap \mathcal{S}^2_{\mathcal{E}}(x) = \mathcal{S}^1_{\mathcal{E}}(x) \subset \mathcal{S}^1_{\mathcal{E}}(y) = \mathcal{S}^1_{\mathcal{E}}(y) \cap \mathcal{S}^2_{\mathcal{E}}(y) = \mathcal{S}_{\mathcal{E}}(y).$$

This proves that $\mathcal{S}_{\mathcal{E}}(x) \subset \mathcal{S}_{\mathcal{E}}(y)$.

□

The next lemma characterizes the elements of the set $\{x \in \mathcal{X} : x_1 \geq \bar{x}_1 \text{ and } x_2 \geq \bar{x}_2\}$. It will be used to prove that statement (2) holds.

Lemma 5. *Assume the utility functions are normalized such that $u_1(0) = u_2(0) = 0$.*[20] *For any $x^* \in \{x \in \mathcal{X} : x_1 \geq \bar{x}_1$ and $x_2 \geq \bar{x}_2\}$ at least one of the following statements is true:*

(a) x^* satisfies $x^*_1 = x^*_2 = \frac{1}{2}$.

(b) There exist $i, j \in \{1, 2\}$ such that $x^*_i < x^*_j$ and

$$\frac{u_i(x^*_i)}{u_i(x^*_i + \Delta)} \geq \frac{u_j(x^*_j)}{u_j(x^*_j + \Delta)} \tag{22}$$

for some $\Delta \in (0, 1 - x^*_j]$.

(c) x^* is the symmetric Nash bargaining solution.

Proof of Lemma 5. Let $x^* \in \{x \in \mathcal{X} : x_1 \geq \bar{x}_1 \text{ and } x_2 \geq \bar{x}_2\}$. Clearly, the statement of the lemma is true if $x^* = (\frac{1}{2}, \frac{1}{2})$. We will prove the lemma for the case where $x^*_1 < x^*_2$. The argument for the case where $x^*_2 < x^*_1$ is analogous.

As $x^* \in \{x \in \mathcal{X} : x_1 \geq \bar{x}_1 \text{ and } x_2 \geq \bar{x}_2\}$ implies that $x^*_1 \geq \bar{x}_1$, it must be that either $x^*_1 > \bar{x}_1$ or $x^*_1 = \bar{x}_1$.

---

[20] If this was not the case, the formula in statement (b) would need to be adjusted.

Consider first the case where $x_1^* > \bar{x}_1$. Note that in this case, by Lemma 2, $x^* \notin \mathcal{D}_1$. However, $x^* \notin \mathcal{D}_1$ implies there must exist $q \in (0,1)$ and $\Delta \in (0,1)$ so that the statement

$$x_2^* + \Delta \leq 1 \text{ and } u_2(x_2^*) \leq q \cdot u_2(x_2^* + \Delta) \tag{23}$$

holds but the statement

$$x_1^* + \Delta \leq 1 \text{ and } u_1(x_1^*) < q \cdot u_1(x_1^* + \Delta) \tag{24}$$

does not. Note now that since we assumed $x_1^* < x_2^*$, the inequality $x_2^* + \Delta \leq 1$ from (23) implies $x_1^* + \Delta \leq 1$ from (24). Thus, if (24) does not hold it must be that

$$u_1(x_1^*) \geq q \cdot u_1(x_1^* + \Delta).$$

Combining the last inequality with the second inequality from (23) we obtain

$$\frac{u_1(x_1^*)}{u_1(x_1^* + \Delta)} \geq \frac{u_2(x_2^*)}{u_2(x_2^* + \Delta)}.$$

Thus, in this case, x^* satisfies condition (b) in the statement of the lemma.

All that remains is to prove the lemma for the case where $x_1^* = \bar{x}_1$. To this end, let x^n be a sequence of allocations such that $\frac{1}{2} > x_1^n > \bar{x}_1$ and $x^n \to \bar{x}_1$. Note that since $x_1^n > \bar{x}_1$, the same reasoning that yielded (22) for x^* with $\frac{1}{2} > x_1^* > \bar{x}_1$ will yield that for each n there exists $\Delta^n \in (0, x_1^n]$ such that

$$\frac{u_1(x_1^n)}{u_1(x_1^n + \Delta^n)} \geq \frac{u_2(x_2^n)}{u_2(x_2^n + \Delta^n)}. \tag{25}$$

Given that $\Delta^n \in [0,1]$ for all n and $[0,1]$ is compact there exists a convergent subsequence Δ^{n_k}. Let $\Delta = \lim_{k\to\infty} \Delta^{n_k} \in [0, \bar{x}_1]$.

If $\Delta > 0$, inequalities (25) together with the fact that u_1 and u_2 are continuous implies that, in the limit,

$$\frac{u_1(x_1^*)}{u_1(x_1^* + \Delta)} \geq \frac{u_2(x_2^*)}{u_2(x_2^* + \Delta)}.$$

Thus, in this case, x^* satisfies condition (b) in the statement of the lemma.

If $\Delta = 0$, then (25) implies

$$\frac{u_1(x_1^*)}{u_1'(x_1^*)} \geq \frac{u_2(x_2^*)}{u_2'(x_2^*)}.$$

Note that if the last inequality is binding, then x^* satisfies condition (c) in the statement of the lemma.[21] On the other hand, if the last inequality is strict then for all sufficiently small positive h, it will be the case that

$$\frac{u_1(x_1^*)}{u_1(x_1^* + h)} \geq \frac{u_2(x_2^*)}{u_2(x_2^* + h)}.$$

Thus, in this case x^* satisfies condition (b) in the statement of the lemma.

□

We are now ready to prove Theorem 1 by showing that (1), (2), and (3) stated at the beginning of this section hold.

Proof of Statement (1). To show that $\mathcal{X}^{\text{easiest}} \subset \{x \in \mathcal{X} : x_1 \geq \bar{x}_1 \text{ and } x_2 \geq \bar{x}_2\}$, assume there is an allocation $x^* \in \mathcal{X}^{\text{easiest}}$ such that $x^* \notin \{x \in \mathcal{X} : x_1 \geq \bar{x}_1 \text{ and } x_2 \geq \bar{x}_2\}$.

If $x^* \notin \{x \in \mathcal{X} : x_1 \geq \bar{x}_1 \text{ and } x_2 \geq \bar{x}_2\}$ then $x_i^* < \bar{x}_i$ holds for some player i. Let $y \in \mathcal{X}$ be an allocation such that $x_i^* < y_i < \bar{x}_i$. Under Lemma 4, for any enforcement technology set $\mathcal{E}$ it must be that

$$\mathcal{S}_{\mathcal{E}}(x^*) \subset \mathcal{S}_{\mathcal{E}}(y). \tag{26}$$

However, $x^* \in \mathcal{X}^{\text{easiest}}$ implies that there is an enforcement technology set $\mathcal{E}$ such that x^* is the easiest to sustain, meaning $\mathcal{S}_{\mathcal{E}}(y) \subsetneq \mathcal{S}_{\mathcal{E}}(x^*)$ holds for any $y \neq x^*$. This contradicts (26).

Proof of Statement (2). We will now show that for any $x^* \in \{x \in \mathcal{X} : x_1 \geq \bar{x}_1 \text{ and } x_2 \geq \bar{x}_2\}$ there exists an enforcement technology set $\mathcal{E}$ and a non-decreasing

[21] The unique allocation satisfying

$$\frac{u_1(x_1^*)}{u_1'(x_1^*)} = \frac{u_2(1 - x_1^*)}{u_2'(1 - x_1^*)}$$

is the Nash bargaining solution as the above equation is the first-order condition for the problem $\max_{x_1 \in [0,1]} u_1(x_1) \cdot u_2(1 - x_1)$.

cost function κ such that x^* is the unique solution to the cost-minimization problem for that enforcement technology set and cost function.

Recall that, in keeping with Remark 1, we can – without loss of generality – restrict attention to the case where $u_i(0) = 0$ for $i \in \{1, 2\}$.

Let $x^* \in \{x \in \mathcal{X} : x_1 \geq \bar{x}_1 \text{ and } x_2 \geq \bar{x}_2\}$. Note that x^* must then satisfy (a), (b), or (c) in Lemma 5. We will consider the three cases separately.

Step 1: Consider first the case where $x^* = (\frac{1}{2}, \frac{1}{2})$, i.e., condition (a) in Lemma 5 is satisfied. For $\varepsilon \in (0, \frac{1}{2})$, define $p^\varepsilon : [0,1]^2 \to [0,1]$ by

$$p^\varepsilon(h, x_i) = \begin{cases} 1 & \text{for } h \in [0,1], x_i \in [\frac{1}{2}, 1] \\ \frac{\varepsilon + x_i - \frac{1}{2}}{\varepsilon} & \text{for } h \in [0,1], x_i \in [\frac{1}{2} - \varepsilon, \frac{1}{2}) \\ 0 & \text{for } h \in [0,1], x_i \in [0, \frac{1}{2} - \varepsilon). \end{cases}$$

Furthermore, let $m_0 : [0,1]^2 \to [0, \infty)$ be defined by

$$m_0(h, x_i) = 0$$

for all $h, x_i \in [0, 1]$.

Let $\mathcal{E}$ be the enforcement technology set given by

$$\mathcal{E} = \{(p^\varepsilon, m_0) : \varepsilon \in (0, \frac{1}{2})\}$$

and $\kappa : \mathcal{E} \to [0, 1]$ be given by

$$\kappa((p^\varepsilon, m_0)) = \varepsilon.$$

Note that $\mathcal{S}_{\mathcal{E}}((\frac{1}{2}, \frac{1}{2})) = \mathcal{E}$. On the other hand, for $x' \in \mathcal{X} - \{(\frac{1}{2}, \frac{1}{2})\}$, it will be the case that $(p^\varepsilon, m_0) \notin \mathcal{S}_{\mathcal{E}}(x')$ whenever $\varepsilon < \frac{|x_1' - x_2'|}{2}$ which implies $\inf_{c \in \mathcal{E}} \kappa(c) \geq \frac{|x_1' - x_2'|}{2}$. This implies that $x^* = (\frac{1}{2}, \frac{1}{2})$ is the unique solution to the cost-minimization problem for the above $\mathcal{E}$ and κ.

Step 2: Next, consider the case where condition (b) in Lemma 5 is satisfied for $i = 1$ and $j = 2$. (The argument in the case where condition (b) in Lemma 5 is satisfied for $i = 2$ and $j = 1$ is analogous.)

Let $m_0 : [0,1]^2 \to [0,\infty)$ again be defined by

$$m_0(h, x_1) = 0$$

for all $h, x_1 \in [0,1]$. Define $p^\varepsilon : [0,1]^2 \to [0,1]$ by

$$p^\varepsilon(h, x_1) = \begin{cases} 1 - \min(\frac{u_1(x_1^*)}{u_1(x_1^*+h)}, \frac{u_2(x_2^*)}{u_2(x_2^*+h)}) & \text{for } h \in [0,1], x_1 \in [x_1^*, 1] \\ \frac{\varepsilon - x_1^* + x_1}{\varepsilon} \cdot (1 - \min(\frac{u_1(x_1^*)}{u_1(x_1^*+h)}, \frac{u_2(x_2^*)}{u_2(x_2^*+h)})) & \text{for } h \in [0,1], x_1 \in [x_1^* - \varepsilon, x_1^*] \\ 0 & \text{for } h \in [0,1], x_1 \in [0, x_1^* - \varepsilon]. \end{cases}$$

Now, consider the enforcement technology set $\mathcal{E}$ defined by

$$\mathcal{E} = \{(p^\varepsilon, m_0) : \varepsilon \in (0, x_1^*]\}.$$

Note that $\mathcal{S}_\mathcal{E}(x^*) = \mathcal{E}$ as

$$\begin{aligned} &(1 - p^\varepsilon(x_i' - x_i^*, x_1^*)) \cdot u_i(x_i') \le \\ &\le \min(\frac{u_1(x_1^*)}{u_1(x_1^* + x_i' - x_i^*)}, \frac{u_2(x_2^*)}{u_2(x_2^* + x_i' - x_i^*)}) \cdot u_i(x_i') \le \\ &\le \frac{u_i(x_i^*)}{u_i(x_i')} \cdot u_i(x_i') = u_i(x_i^*), \end{aligned}$$

implies that $(p^\varepsilon, m_0) \in \mathcal{S}_\mathcal{E}(x^*)$ for all $\varepsilon \in (0, x_1^*]$.

Let $\kappa : \mathcal{E} \to [0,1]$ be given by $\kappa((p^\varepsilon, m_0)) = \varepsilon$. The fact that $(p^\varepsilon, m_0) \in \mathcal{S}_\mathcal{E}(x^*)$ for all $\varepsilon \in (0, x_1^*]$, implies that $\inf_{c \in \mathcal{S}_\mathcal{E}(x^*)} \kappa(c) = 0$.

Note that for any x with $x_1 < x_1^*$ it is the case that $(p^\varepsilon, m_0) \notin \mathcal{S}_\mathcal{E}(x)$ for $\varepsilon < x_1^* - x_1$. This implies that for any x with $x_1 < x_1^*$, $\inf_{c \in \mathcal{S}_\mathcal{E}(x)} \kappa(c) \ge x_1^* - x_1$.

We will now show that for any x with $x_1 > x_1^*$ (and therefore $x_2 < x_2^*$), it is the case that $(p^\varepsilon, m_0) \notin \mathcal{S}_\mathcal{E}(x)$.

To see that this indeed true, recall that in this step we assume that x^* satisfies statement (b) in Lemma 5 for $i = 1$ and $j = 2$. Let $\Delta \in (0, 1 - x_2^*]$ be such that equation (22) from statement (b) in Lemma 5 is satisfied. Note that for $x' =$

$(x_1 - \Delta, x_2 + \Delta)$ we therefore have

$$(1 - p^\varepsilon(x_2' - x_2, x_2)) \cdot u_2(x_2') \geq$$

$$\geq \min\left(\frac{u_1(x_1^*)}{u_1(x_1^* + \Delta)}, \frac{u_2(x_2^*)}{u_2(x_2^* + \Delta)}\right) \cdot u_2(x_2') =$$

$$= \frac{u_2(x_2^*)}{u_2(x_2^* + \Delta)} \cdot u_2(x_2') = \frac{u_2(x_2^*)}{u_2(x_2^* + \Delta)} \cdot u_2(x_2 + \Delta) > u_2(x_2),$$

where the first inequality follows from the definition of p^ε, the first equality follows from inequality (22), and the last inequality holds as $\frac{u_2(z_2)}{u_2(z_2+\Delta)}$ is increasing in z_2.[22] Given that $\mathcal{S}_\mathcal{E}(x^*) = \mathcal{E}$ and $\mathcal{S}_\mathcal{E}(x) \subsetneq \mathcal{E}$ for $x \neq x^*$, we have shown that x^* is the easiest to sustain for the enforcement technology set $\mathcal{E}$.

Step 3: Next, consider the case where x^* satisfies condition (c) in Lemma 5, i.e., x^* is the Nash bargaining solution. For this case, let $\mathcal{E}$ be the enforcement technology set given defined in Section 3.2 and $\kappa : \mathcal{E} \to [0, 1]$ be given by $\kappa((p, m_0)) = D_1 p(0, 0)$. Given the analysis in Section 3.2 it is trivial to verify that x^* is the unique solution to the cost-minimization problem for this $\mathcal{E}$ and κ.

Proof of Statement (3). To show that $\mathcal{X}^{\text{cost-min}} \subset \mathcal{X}^{\text{easiest}}$, assume $x \in \mathcal{X}$ is the unique solution to the cost-minimization problem (3) for some $\mathcal{E}$ and κ. Let $\mathcal{E}' = \mathcal{E} \cap \mathcal{S}_\mathcal{E}(x)$. We claim that x is the easiest to sustain for the enforcement technology set $\mathcal{E}'$, i.e., for any $y \in \mathcal{X}$ s.t. $y \neq x$ it is the case that $\mathcal{S}_{\mathcal{E}'}(y) \subsetneq \mathcal{S}_{\mathcal{E}'}(x)$. To see why this is so, note that $\mathcal{S}_{\mathcal{E}'}(y) \subset \mathcal{S}_{\mathcal{E}'}(x)$ follows from the definition of $\mathcal{E}'$ and that it cannot be that $\mathcal{S}_{\mathcal{E}'}(y) = \mathcal{S}_{\mathcal{E}'}(x)$, as this would imply $\mathcal{S}_\mathcal{E}(x) \subset \mathcal{S}_\mathcal{E}(y)$, which would contradict that x is the unique solution to (3).

6.2. **Proof of Theorem 2.** Let $\hat{\mathcal{X}}^{\text{cost-min}}$ be the set of allocations $x \in \mathcal{X}$ satisfying statement (i) in Theorem 1 where the requirement that the enforcement technology is regular has been dropped. Similarly, let $\hat{\mathcal{X}}^{\text{easiest}}$ be the set of allocations $x \in \mathcal{X}$

[22] To prove that $\frac{u_2(z_2)}{u_2(z_2+\Delta)}$ is increasing in z_2 it is enough to show that $\frac{u_2(z_2+\Delta)}{u_2(z_2)}$ is decreasing in z_2 but $\frac{u_2(z_2+\Delta)}{u_2(z_2)} = 1 + \frac{u_2(z_2+\Delta)-u_2(z_2)}{u_2(z_2)}$, $u_2(z_2 + \Delta) - u_2(z_2)$ is decreasing in z_2 (as u_2 is a concave function) and $u_2(z_2)$ is increasing in z_2 (as u_2 is an increasing function).

satisfying statement (2) in Theorem 1 where the requirement that the enforcement technology is regular has been dropped.

To prove Theorem 2 it is enough to show that:

(1') $\hat{\mathcal{X}}^{\text{easiest}} \subset \{x \in \mathcal{X} : x_1 \geq \bar{x}_1 \text{ and } x_2 \geq \bar{x}_2\}$,

(2') $\{x \in \mathcal{X} : x_1 \geq \bar{x}_1 \text{ and } x_2 \geq \bar{x}_2\} \subset \hat{\mathcal{X}}^{\text{cost-min}}$, and

(3') $\hat{\mathcal{X}}^{\text{cost-min}} \subset \hat{\mathcal{X}}^{\text{easiest}}$.

To prove (1') note that the proof of statement (1) in the proof of Theorem 1 did not use the fact that the considered enforcement technology sets were regular and thus the same argument yields that statement (1') holds. To prove (2') note that $\mathcal{X}^{\text{cost-min}} \subset \hat{\mathcal{X}}^{\text{cost-min}}$. Thus, (2') follows from (2) proven in the proof of Theorem 1. To prove (3') note again that the argument used in the proof of Theorem 1 to prove (3) did not use the fact that the considered enforcement sets were regular and thus can be immediately generalized to show that (3) holds.

6.3. **Proof of Proposition 3.** Let us start by noticing that the values $\bar{y}_1$ and $\bar{y}_2$ are indeed well-defined.

Lemma 6. *Let $i, j \in \{1, 2\}$ such that $i \neq j$. Then the equation*

$$\frac{\frac{u_i(2\bar{y}_i) - u_i(\bar{y}_i)}{\bar{y}_i}}{u_i(\bar{y}_i) - u_i(0)} = \frac{u_j'(1-\bar{y}_i)}{u_j(1-\bar{y}_i) - u_j(0)}$$

has a unique solution $\bar{y}_i$. Moreover, if x^{NBS} is the Nash bargaining solution, $\bar{y}_i \leq x_i^{NBS}$ holds.

Proof of Lemma 6. Consider the equation in the statement of the lemma. Note that, for $\bar{y}_i \to 0$, the right-hand side of the equation goes to infinity[23] and the left-hand side of the equation converges to $u_j'(1)/(u_j(1) - u_j(0))$. Similarly, for $\bar{y}_i \to 1$, the right-hand side converges to infinity and the left-hand side converges to some real number. Since the left-hand side and the right-hand side are continuous in $\bar{y}_i$ for $\bar{y}_i \in (0, 1)$, this implies that the equation in the lemma has a solution.

[23]This follows from the observation that $\frac{u_i(2\bar{y}_i) - u_i(\bar{y}_i)}{\bar{y}_i}$ converges to $u_i'(0)$ as $\bar{y}_i \to 0$.

The uniqueness of the solution follows from the fact that the right-hand side of the equation in the lemma increases in $\bar{y}_i$ and the left hand side decreases in $\bar{y}_i$. Indeed, the fact that the right-hand side increases in $\bar{y}_i$ follows immediately from the fact that u_j is increasing and concave. The fact that the left-hand side decreases in $\bar{y}_i$ follows from the fact that $u_i(\bar{y}_i) - u_i(0)$ increases in $\bar{y}_i$ and the observation that $\frac{u_i(2\bar{y}_i)-u_i(\bar{y}_i)}{\bar{y}_i} = \int_{y_i'=\bar{y}_i}^{2\cdot\bar{y}_i} \frac{1}{\bar{y}_i} \cdot u_i'(y_i') \cdot dy_i'$ is decreasing in $\bar{y}_i$.

The fact that $\bar{y}_i \leq x_i^{NBS}$ follows from the above monotonicity results together with the observation that $\frac{u_i(2\bar{y}_i)-u_i(\bar{y}_i)}{\bar{y}_i} < u_i'(\bar{y}_i)$ and the fact that x_i^{NBS} solves

$$\frac{u_i'(x_i^{NBS})}{u_i(x_i^{NBS}) - u_i(0)} = \frac{u_j'(1-x_i^{NBS})}{u_j(1-x_i^{NBS}) - u_j(0)}.$$

□

Proof of Proposition 3. The fact that $\mathcal{X}^* = \{x \in \mathcal{X} : x_1 \geq \bar{x}_1 \text{ and } x_2 \geq \bar{x}_2\}$ is a subset of $\mathcal{Y}^*$ now follows from Lemma 5 and Lemma 6. To see that this is indeed the case, assume without loss of generality that u_1 and u_2 have been normalized so that $u_1(0) = u_2(0) = 0$ and $u_1(1) = u_2 = 1$.[24] Let $x^* \in \mathcal{X}^*$. Since $x^* \in \mathcal{X}^*$, Lemma 5 tells us that x^* must satisfy at least one of the conditions (a), (b), or (c) in Lemma 5. Note that if x^* satisfies conditions (a) or (c), Lemma 6 immediately implies $x^* \in \mathcal{Y}^*$. Consider therefore the case where x^* satisfies condition (b), meaning there exists $i, j \in \{1, 2\}$ with the property that $x_i^* < x_j^*$ and (22) holds for some $\Delta \in (0, 1 - x_j^*] = (0, x_i^*]$. Rearranging terms and subtracting 1 from both sides of the inequality note that (22) is equivalent to

$$\frac{u_i(x_i^* + \Delta) - u_i(x_i^*)}{u_i(x_i^*)} \leq \frac{u_j(x_j^* + \Delta) - u_j(x_j^*)}{u_j(x_j^*)}.$$

The fact that u_i is concave and $\Delta \leq x_i^*$ implies

$$\frac{u_i(2 \cdot x_i^*) - u_i(x_i^*)}{x_i^*} \leq \frac{u_i(x_i^* + \Delta) - u_i(x_i^*)}{\Delta}.$$

[24] This is without loss of generality as neither the set $\mathcal{X}^*$ nor the definition of $\bar{y}_1$ and $\bar{y}_2$ are affected by positive affine transformations of the utility functions.

The fact that u_j is concave implies

$$\frac{u_j(x_j^* + \Delta) - u_j(x_j^*)}{\Delta} \le u_j'(x_j).$$

Combining the last three inequalities and using $x_j^* = 1 - x_i^*$ yields

$$\frac{\frac{u_i(2\cdot x_i^*) - u_i(x_i^*)}{x_i^*}}{u_i(x_i^*)} \le \frac{u_j'(1 - x_i^*)}{u_j(1 - x_i^*)}.$$

In the proof of Lemma 6 we have already argued that the left-hand side of the above inequality decreases in x_i^* and the right-hand side increases in x_i^*. This implies $x_i^* \ge \bar{y}_i$. However, $x_i^* < x_j^*$ implies $x_j^* \ge \frac{1}{2}$. Thus, $x_i^* \ge \min(\frac{1}{2}, \bar{y}_i)$ and $x_j^* \ge \min(\frac{1}{2}, \bar{y}_j)$ which means that $x^* \in \mathcal{Y}^*$.

$\square$

6.4. **Proof of Theorem 3.** Define $\mathcal{X}^{\text{undom}}$ as in Subsection 4.5, see the paragraph that follows Theorem 3. As was discussed in Subsection 4.5, in the paragraphs following Theorem 3, to prove the theorem it is enough to show that $\mathcal{X}^{\text{undom}} - \mathcal{X}^{\text{easiest}}$ is empty. We will show $\mathcal{X}^{\text{undom}} - \mathcal{X}^{\text{easiest}} = \emptyset$ by showing that $x \notin \mathcal{X}^{\text{easiest}}$ implies $x \notin \mathcal{X}^{\text{undom}}$.

Assume x is an allocation such that $x \notin \mathcal{X}^{\text{easiest}}$. By Theorem 1, $\mathcal{X}^{\text{easiest}} = \{x \in \mathcal{X} : x_1 \ge \bar{x}_1 \text{ and } x_2 \ge \bar{x}_2\}$, where $\bar{x}_1$ and $\bar{x}_2$ are defined in as in Theorem 1. Thus, $x \notin \mathcal{X}^{\text{easiest}}$ implies $x \notin \{x \in \mathcal{X} : x_1 \ge \bar{x}_1 \text{ and } x_2 \ge \bar{x}_2\}$.

Let $i \in \{1, 2\}$ be such that $x_i < \bar{x}_i$. Let $y \in \mathcal{X}$ be such that $x_i < y_i < \bar{x}_i$.

In accord with Lemma 4, $\mathcal{S}_{\mathcal{E}}(x) \subset \mathcal{S}_{\mathcal{E}}(y)$ holds for any regular $\mathcal{E}$.

Let (p, m) be given by $m \equiv 0$, $p(h, x_i') = 0$ for $h \in [0, 1]$ and $x_i' \in [0, x_i]$, $p(h, x_i') = \frac{x_i' - x_i}{y_i - x_i}$ for $h \in [0, 1]$ and $x_i' \in [x_i, y_i]$, and $p(h, x_i') = 1$ for $h \in [0, 1]$ and $x_i' \in [y_i, 1]$. Let $\hat{\mathcal{E}} = \{(p, m)\}$ and note that $\mathcal{S}_{\hat{\mathcal{E}}}(y) = \hat{\mathcal{E}}$ and $\mathcal{S}_{\hat{\mathcal{E}}}(x) = \emptyset$. Since $\mathcal{S}_{\mathcal{E}}(x) \subset \mathcal{S}_{\mathcal{E}}(y)$ holds for any regular $\mathcal{E}$ and $\mathcal{S}_{\mathcal{E}}(x) \subsetneq \mathcal{S}_{\mathcal{E}}(y)$ holds for $\mathcal{E} = \hat{\mathcal{E}}$, x is dominated by y and, therefore, $x \notin \mathcal{X}^{\text{undom}}$.

References

[1] Abreu, Dilip and Faruk Gul (2000) "Bargaining and Reputation," *Econometrica*, 68, 85 – 117

[2] Arrow, Kenneth (1971) "Political and Economic Evaluation of Social Effects and Externalities," *Frontiers of Quantitative Economics.*, Amsterdam: North-Holland, p. 3-25

[3] Crawford, Vincent P. (1982) "A Theory of Disagreement in Bargaining" *Econometrica*, 50:3, 607 – 637

[4] Compte, Olivier and Philippe Jehiel (2010) "The coalitional Nash Bargaining Solution," *Econometrica*, 78, 1593 – 1623

[5] Kalai, Ehud and Meir Smorodinsky (1975) "Other Solutions to Nash's Bargaining Problem," *Econometrica*, 43, 513 – 518

[6] Myerson, Roger B. (1991) "Game Theory: Analysis of Conflict," *Harvard University Press*

[7] Perry, Motty and Philip Reny (1994) "A Nooncooperative View of Coalition Formation and the Core," *Econometrica*, 62, 795 – 817

[8] Nash, John F. (1950) "The Bargaining Problem," *Econometrica*, 18, 155 – 162

[9] Nash, John F. (1953) "Two-Person Cooperative Games," *Econometrica*, 21, 128 – 140

[10] Rubinstein, Ariel (1982) "Perfect Equilibrium in a Bargaining Model," *Econometrica*, 50, 97 – 109

[11] Rubinstein, Ariel, Zvi Safra, and William Thomson (1992) "On the Interpretation of the Nash Bargaining Solution and Its Extensions to Non-Expected Utility Preferences," *Econometrica*, 60, 1171 – 1186

[12] Samuelson, Larry (2004): "Information-Based Relative Consumption Effects," *Econometrica* 72:1, 93-118.

[13] Samuelson, Larry and Jeroen Swinkels (2006): "Information, evolution, and utility," *Theoretical Economics* 1, 119-142.

[14] Young, H. Peyton (1993): "An Evolutionary Model of Bargaining," *Journal of Economic Theory* 59:1, 145-168.