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Rhetoric in legislative bargaining with asymmetric information

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We analyze a three-player legislative bargaining game over an ideological and a distributive decision. Legislators are privately informed about their ideological intensities, i.e., the weight placed on the ideological decision relative to the weight placed on the distributive decision. Communication takes place before a proposal is offered and majority rule voting determines the outcome. We show that it is not possible for all legislators to communicate informatively. In particular, the legislator who is ideologically more distant from the proposer cannot communicate informatively, but the closer legislator may communicate whether he would "compromise" or "fight" on ideology. Surprisingly, the proposer may be worse off when bargaining with two legislators (under majority rule) than with one (who has veto power), because competition between the legislators may result in less information conveyed in equilibrium. Despite separable preferences, the proposer is always better off making proposals for the two dimensions together.

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JEL classification. C78, D72, D82, D83.

1. Introduction

Legislative policy-making typically involves speeches and demands by legislators that may shape the proposals made by the leadership. For example, in the 2010 health care overhaul in the United States, one version of the Senate bill included $100 million in Medicaid funding for Nebraska and restrictions on abortion coverage in exchange for the vote of Nebraska Senator Ben Nelson. As another example, consider the threat in 2009 by seven members of the U.S. Senate Budget Committee to withhold their support for legislation to raise the debt ceiling unless a commission to recommend cuts to Medicare and Social Security was approved.[1] Would these senators indeed have let the United

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[1] See http://thehill.com/homenews/senate/67293-sens-squeeze-speaker-over-commission.

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States default on its debt or was their demand just a bluff? More generally, what are the patterns of demands in legislative policy-making? How much information do they convey? Do they influence the nature of the proposed bills? Who gets private benefits and what kind of policies are chosen under the ultimately accepted bills?

To answer these questions, it is necessary to have a legislative bargaining model in which legislators make demands before the proposal of the bills. One approach is to assume that the demands serve as a commitment device, that is, the legislators refuse any offer that does not meet their demands.[2] While this approach offers interesting insights into some of the questions above, it relies on the strong assumption that legislators commit to their demands.[3] In this paper, we offer a different approach that allows legislators to make speeches without commitment as to how they will cast their votes. The premise of our approach is that only individual legislators know which bills they prefer to the status quo. So even if the legislators do not necessarily carry out their threats, their demands may be meaningful rhetoric in conveying private information and dispelling some uncertainty in the bargaining process.

We model rhetoric as cheap-talk messages as in Matthews (1989). In our model, (i) three legislators bargain over an ideological and a distributive decision; (ii) one of the legislators, called the chair, formulates a proposal; (iii) each legislator other than the chair is privately informed about his own preferences; (iv) communication takes place before a proposal is offered; (v) majority rule voting determines whether the proposal is implemented.

We assume each legislator's position on a unidimensional ideological spectrum is publicly known, but his ideological intensity (the weight he places on the ideological dimension relative to the distributive dimension) is his private information. As such, the chair is unsure how much transfer she has to offer to a legislator to gain his support for a policy decision, but she can use the messages sent in the communication stage to make inferences about his ideological intensity (which we call his type). We focus on a class of equilibria called *simple connected equilibria* in which the types who send the same message form an interval (*connected*), and the proposal does not depend on the message of a legislator if he receives no transfer (*simple*). We find the restriction to simple equilibrium reasonable because we show that if a legislator receives no transfer from a proposal, then he will surely reject the proposal in equilibrium. As a result, if two equilibrium proposals both give no transfer to a legislator, then the proposals depend only on the chair's belief about the other legislator's type, which has nothing to do with the message sent by the legislator who receives no transfer, justifying the simple restriction.

We show that in any simple connected equilibrium, (i) at most one legislator's messages convey some information about his preferences (Proposition 4(i)). (ii) In particular, if the legislator whose position is closer to the chair's wants to move the policy in the same direction as the chair does, then it is impossible for the other legislator (whose position is further away from the chair's) to be informative (Proposition 4(ii)). (iii) Although

[2]This is the approach taken by Morelli (1999) in a complete information framework. He does not explicitly model the proposal-making and the voting stages. As such, the commitment assumption is implicit.

[3]Politicians often make empty threats. See, for example, http://thehill.com/homenews/news/14312-gopsays-it-can-call-reids-bluffs.

the closer legislator may be informative, even he can convey only limited information (Proposition 5). Specifically, he sends a "fight" message when he places a relatively high weight on the ideological dimension, and the chair responds with a proposal that involves minimum policy change and gives neither legislator any private benefit since the message indicates that there is no room for making a deal. When he places a relatively low weight on the ideological dimension, he sends a "compromise" message and the chair responds by offering some private benefit in exchange for moving the policy closer to her own ideal. In contrast to the classic Crawford and Sobel (1982) model of cheap talk in which the sender conveys increasingly more precise information when the players' interests become closer, here, it is impossible for even the closer legislator to convey more than whether he will compromise or fight, no matter how close his position is to the chair's.

Surprisingly, bargaining with two legislators under majority rule may make the chair worse off than if she bargains with only one legislator (who can veto a bill). Under complete information, the chair is clearly better off when bargaining with two legislators instead of one because her bargaining position is improved. Under asymmetric information, however, the number of legislators also affects the amount of information transmission. In particular, increased competition may undermine a legislator's incentive to send the fight message, resulting in less information transmitted in equilibrium, and this hurts the chair.

Since the players bargain over both an ideological dimension and a distributive dimension, a natural question is whether it is better to bundle the two issues in one bill or negotiate over them separately. In our model, bundling always benefits the chair because she can exploit the differences in the other legislators' trade-offs between the two dimensions, and use private benefit as an instrument to make deals on policy changes that she wants to implement. This result, however, depends on the nature of uncertainty regarding preferences. In a related paper (Chen and Eraslan 2013), we show that bundling may result in informational loss when ideological positions are private information, and in that case, bundling might hurt the chair.

Before turning to the description of our model, we briefly discuss the related literature. Starting with the seminal work of Baron and Ferejohn (1989), legislative bargaining models have become a staple of political economy and have been used in numerous applications. Like our paper, some papers in the literature include an ideological dimension and a distributive dimension (e.g., Austen-Smith and Banks 1988, Banks and Duggan 2000, Jackson and Moselle 2002, and Diermeier and Merlo 2000), but all these papers take the form of sequential offers and do not incorporate demands. A smaller strand of literature, notably Morelli (1999), instead models the legislative process as a sequential demand game where the legislators commit to their demands.[4] With the exceptions of Tsai (2009), and Tsai and Yang (2010a, 2010b), who do not model demands, all of these papers assume complete information.

The cheap-talk literature has largely progressed in parallel to the bargaining literature. Exceptions are Farrell and Gibbons (1989), Matthews (1989), and Matthews and

[4]See also Vidal-Puga (2004), Montero and Vidal-Puga (2007), and Breitmoser (2009).

Postlewaite (1989). Of these, Matthews (1989) is the most closely related. Our model differs from his by having multiple legislators (rather than one) who are privately informed about their ideological intensities (rather than ideological positions); moreover, in our model,the players bargain over an ideological and a distributive decision, whereas in Matthews (1989), they bargain over an ideological decision only. Our paper is also related to cheap-talk games with multiple senders (e.g., Gilligan and Krehbiel 1989, Austen-Smith 1993, Krishna and Morgan 2001a, 2001b, Battaglini 2002, and Ambrus and Takahashi 2008). Our framework differs from these papers because it has voting over the receiver's proposal and also incorporates a distributive dimension.

In the next section, we describe our model. We first consider the complete information model as a benchmark in Section 3. We then study the bargaining game in which the legislators' ideological intensities are private information. In Section 4, we analyze the simpler game with only one legislator (other than the chair) and then move on to analyze the game with two legislators in Section 5. We discuss extensions and generalizations in Section 6.

2. Model

Three legislators play a three-stage game to collectively decide on an outcome that consists of an ideological component and a distributive component. For example, the legislators decide on the level of environmental regulation and the distribution of government spending across districts. Legislator 0 makes the proposal.[5] From now on, we simply refer to legislator 0 as the chair and use the term "legislator" to refer to the other two players. Let $z = (y; x)$, where y is an ideological decision and $x = (x_0, x_1, x_2)$ is a distributive decision. The set of feasible ideological decisions is $Y = \mathbb{R}$ and the set of feasible distributions is $X = \{x \in \mathbb{R}^3 : \sum_{i=0}^{2} x_i \leq c, x_1 \geq 0, x_2 \geq 0\}$, where x_i denotes the private benefit of player i and $c \geq 0$ is the size of the surplus available for division. For $i = 1, 2$, we say that proposal $(y; x)$ *includes* legislator i if $x_i > 0$ and *excludes* legislator i if $x_i = 0$.[6] The status quo allocation is $s = (\tilde{y}; \tilde{x})$, where $\tilde{y} \in Y$ and $\tilde{x} = (0, 0, 0)$.[7]

The payoff of each player $i = 0, 1, 2$ depends on the ideological decision and his/her private benefit. We assume that the players' preferences are separable over the two dimensions. Specifically, player i has a quasilinear von Neumann–Morgenstern utility function given by

$$u_i(z, \theta_i, \hat{y}_i) = x_i + \theta_i v(y, \hat{y}_i),$$

where $z = (y; x)$ is the outcome, $\hat{y}_i \in Y$ is player i's ideal point (ideological position), and $\theta_i > 0$ is the weight that player i places on his/her payoff from the ideological decision relative to the distributive decision. The marginal rate of substitution,

[5]We use "she" as the pronoun for the proposer and "he" as the pronoun for legislators 1 and 2.

[6]In the remainder of the paper, when we use i and j to index the legislators, we sometimes omit the quantifiers $i = 1, 2$ or $j = 1, 2$. When we refer to both legislator i and legislator j, we implicitly assume $j \neq i$.

[7]The assumption that $\tilde{x} = (0, 0, 0)$, together with the definition of X, implies that the total surplus for reaching an agreement is nonnegative, legislator 1's and legislator 2's status quo private benefits are the same, and the chair's proposal cannot offer private benefits lower than his status quo for either legislator 1 or 2.

$(\partial u_i/\partial y)/(\partial u_i/\partial x_i) = \theta_i(\partial v/\partial y)$, measures player i's preference for ideology relative to private benefit. With fixed $\hat{y}_i$, its absolute value is increasing in θ_i, which we call legislator i's ideological intensity parameter.

Legislator $i = 1, 2$ privately observes the realization of θ_i, called his type. The set of possible types of legislator i is $\mathbb{R}_+$. Legislator i's type θ_i is a random variable with a continuous distribution function F_i that has support on $\Theta_i = [\underline{\theta}_i, \overline{\theta}_i]$, and the legislators' types are independently distributed. Although θ_i is legislator i's private information, its distribution and other aspects of his payoff function, including $\hat{y}_i$, are common knowledge. In the remainder of the paper, $\hat{y}_i$ is fixed and we use $u_i(z, \theta_i)$ to denote legislator i's payoff from outcome z when his type is θ_i.

For simplicity we assume the chair's preferences are commonly known. Without loss of generality, assume $\hat{y}_0 < \tilde{y}$ so that the chair would like to move the policy to the left of the status quo. To simplify notation, we write $u_0(z) = x_0 + \theta_0 v(y, \hat{y}_0)$ as the chair's payoff from z.

We make the following assumptions on v: (i) v is twice differentiable; (ii) for any $\hat{y}_i \in Y$, $v_{11}(y, \hat{y}_i) < 0$ for all $y \in Y$ (which implies that v is concave in y), and $v(\cdot, \hat{y}_i)$ reaches its maximum at $\hat{y}_i$; (iii) v satisfies the single-crossing property in $(y, \hat{y}_i)$, i.e., for all $y, y', \hat{y}_i, \hat{y}'_i \in Y$ such that $y' > y$ and $\hat{y}'_i > \hat{y}_i$, if $v(y', \hat{y}_i) \geq v(y, \hat{y}_i)$, then $v(y', \hat{y}'_i) > v(y, \hat{y}'_i)$. This property says that if legislator i whose ideal point is $\hat{y}_i$ weakly prefers y' to y, where y' is to the right of y, then any legislator whose ideal point is to the right of $\hat{y}_i$ strictly prefers y' to y. Note that the familiar quadratic-loss function, $v(y, \hat{y}_i) = -(y - \hat{y}_i)^2$, satisfies all of these assumptions.

The bargaining game has three stages. In stage 1, each legislator $i = 1, 2$ observes his type θ_i and sends a message to the chair simultaneously.[8] In stage 2, the chair observes the messages and makes a proposal in $Y \times X$. In stage 3, the players vote on the proposal under majority rule. Without loss of generality, we assume that the chair votes for the proposal. So a proposal passes if at least one of legislators 1 and 2 votes for it. Otherwise, the status quo s prevails.

If $v(\hat{y}_0, \hat{y}_i) \geq v(\tilde{y}, \hat{y}_i)$ for some legislator i, then there is a legislator who weakly prefers the chair's ideal policy to the status quo policy and the chair's problem is trivial: she proposes her ideal policy and keeps all the private benefit herself. From now on, we assume $v(\hat{y}_0, \hat{y}_i) < v(\tilde{y}, \hat{y}_i)$ for $i = 1, 2$. Note that since $\hat{y}_0 < \tilde{y}$, this implies that $\hat{y}_0 < \hat{y}_i$ for $i = 1, 2$.

The set of allowed messages for legislator i, denoted by M_i, is a finite set that has more than two elements. The messages have no literal meanings (we discuss their equilibrium meanings later); they are also "cheap talk" since they do not affect the players' payoffs directly. The assumption that M_i is finite rules out the possibility of separating equilibria, but we show that separating equilibria are not possible even if M_i's are infinite.

A strategy for legislator i consists of a message rule in the first stage and an acceptance rule in the third stage. A message rule $\mu_i : \Theta_i \to M_i$ specifies the message legislator i sends as a function of his type. An acceptance rule $\gamma_i : Y \times X \times \Theta_i \to \{0, 1\}$ specifies

[8]The messages can be either private or public. Since condition (E1) in the upcoming definition of equilibrium requires that each legislator votes for a proposal if and only if he weakly prefers that proposal to the status quo, our results do not depend on whether the messages are private or public.

how legislator i votes as a function of his type: type θ_i accepts a proposal z if $\gamma_i(z, \theta_i) = 1$ and rejects it if $\gamma_i(z, \theta_i) = 0$.[9] The strategy set for legislator i consists of pairs of measurable functions (μ_i, γ_i) that satisfy these properties. The chair's strategy set consists of all proposal rules $\pi : M_1 \times M_2 \to Y \times X$, where $\pi(m_1, m_2)$ is the proposal she offers when receiving (m_1, m_2). We focus on pure strategies and discuss conditions under which it is not restrictive to disallow mixed strategies later.

Fix a strategy profile (μ, γ, π). Say that a *message profile* $m = (m_1, m_2)$ *induces* proposal z if $\pi(m) = z$. Proposal z is *elicited* by type θ_i if it is induced by m with $m_i = \mu_i(\theta_i)$ and $\{\theta_j : \mu_j(\theta_j) = m_j\} \neq \varnothing$. If z is induced by m, then legislator i is *pivotal with respect to* z if $\gamma_j(z, \theta_j) = 0$ for all θ_j such that $\mu_j(\theta_j) = m_j$ and *nonpivotal with respect to* z otherwise.

To define an equilibrium for this game, let $\beta_i(z|m_i)$ denote the probabilistic belief of the chair that legislator i votes to accept proposal z conditional on receiving message m_i. Given the strategy (μ_i, γ_i) of legislator i, β_i is derived by Bayes' rule whenever possible. That is,

$$\beta_i(z|m_i) = \int_{\{\theta_i:\mu_i(\theta_i)=m_i\}} \gamma_i(z, \theta_i)\, dF_i(\theta_i) \Big/ \int_{\{\theta_i:\mu_i(\theta_i)=m_i\}} dF_i(\theta_i)$$

if $\int_{\{\theta_i:\mu_i(\theta_i)=m_i\}} dF_i(\theta_i) > 0$.

Definition 1. An equilibrium is a strategy profile (μ, γ, π) such that the following conditions hold for all $i \neq 0$, $\theta_i \in \Theta_i$, $y \in Y$, $x \in X$, and $m \in M_1 \times M_2$:

(E1) If $u_i(z, \theta_i) \geq u_i(s, \theta_i)$, then $\gamma(z, \theta_i) = 1$; otherwise, $\gamma_i(z, \theta_i) = 0$.

(E2) We have $\pi(m) \in \arg\max_{z' \in Y \times X} u_0(z')\beta(z'|m) + u_0(s)(1 - \beta(z'|m))$, where

$$\beta(z'|m) = 1 - (1 - \beta_1(z'|m_1))(1 - \beta_2(z'|m_2))$$

is the conditional probability that z' is accepted.

(E3) If $\mu_i(\theta_i) = m_i$, then $m_i \in \arg\max_{m_i'} V_i(m_i', \theta_i)$, where

$$\begin{aligned} V_i(m_i', \theta_i) = \int_{\Theta_j} \big[&\gamma_j(\pi(m_i', \mu_j(\theta_j)), \theta_j) u_i(\pi(m_i', \mu_j(\theta_j)), \theta_i) \\ &+ \big(1 - \gamma_j(\pi(m_i', \mu_j(\theta_j)), \theta_j)\big) \\ &\times \max\{u_i(\pi(m_i', \mu_j(\theta_j)), \theta_i), u_i(s, \theta_i)\}\big]\, dF_j(\theta_j). \end{aligned}$$

Condition (E1) requires each legislator to accept a proposal if and only if he weakly prefers it to the status quo.[10] Condition (E2) requires that equilibrium proposals maximize the chair's payoff and that her belief is consistent with Bayes' rule. Condition (E3)

[9]Technically a legislator's acceptance rule can depend on his message when it is private and can depend on both legislators' messages when they are public. However, our equilibrium condition says that legislator i accepts a proposal if and only if he weakly prefers it to the status quo, independent of the message he sent. As such, we suppress the dependence of γ_i on m_i.

[10]Condition (E1) strengthens the requirement of perfect Bayesian equilibrium (PBE) and is the only difference between our equilibrium solution concept and PBE. In particular, (E1) rules out the (weakly dominated) acceptance rule of accepting any proposal because a legislator expects that the other legislator accepts any proposal. Condition (E1) also assumes that a legislator accepts z when indifferent between z

requires that a legislator elicits only his most preferred distribution of proposals, incorporating the acceptance rules.

For expositional simplicity, from now on we assume that in equilibrium, if $\beta(z|m) = 0$, then $\pi(m) \neq z$. This means that if a proposal is rejected with probability 1, then the chair does not propose it.[11]

Say that a proposal z is *elicited in the equilibrium* (μ, γ, π) if there exists $(\theta_1, \theta_2) \in \Theta_1 \times \Theta_2$ such that $z = \pi(\mu_1(\theta_1), \mu_2(\theta_2))$. For any strategy profile (μ, γ, π), denote by $\phi^{\mu,\gamma,\pi}(\theta_1, \theta_2)$ the *outcome* for (θ_1, θ_2). Specifically, $\phi^{\mu,\gamma,\pi}(\theta_1, \theta_2) = \pi(\mu_1(\theta_1), \mu_2(\theta_2))$ if $\gamma_i(\pi(\mu_1(\theta_1), \mu_2(\theta_2)), \theta_i) = 1$ for at least one of $i = 1, 2$ and $\phi^{\mu,\gamma,\pi}(\theta_1, \theta_2) = s$ otherwise. Say that two equilibria (μ, γ, π) and (μ', γ', π') are *outcome equivalent* if $\phi^{\mu,\gamma,\pi}(\theta_1, \theta_2) = \phi^{\mu',\gamma',\pi'}(\theta_1, \theta_2)$ for almost all $(\theta_1, \theta_2) \in \Theta_1 \times \Theta_2$.

A *babbling equilibrium* is an equilibrium (μ, γ, π) in which $\mu_i(\theta_i) = \mu_i(\theta_i')$ for all $\theta_i, \theta_i' \in \Theta_i$, $i = 1, 2$ (all types of legislator i send the same message) and $\pi(m) = \pi(m')$ for all $m, m' \in M_1 \times M_2$ (the chair responds to all message profiles with the same proposal). As is standard in cheap-talk models, a babbling equilibrium always exists.

3. Benchmark: Complete information

We start by analyzing the benchmark game of complete information where θ_i is common knowledge. Since there is no private information, the legislators' messages are irrelevant for the chair's belief and her proposal. The modifications of the players' strategies and equilibrium conditions are straightforward and omitted. We next characterize the chair's equilibrium proposal.

A useful piece of notation is $e(\hat{y}_i) = \min\{y : v(y, \hat{y}_i) = v(\tilde{y}, \hat{y}_i)\}$, the left-most policy y that makes legislator i indifferent between y and $\tilde{y}$. Recall that $v(\hat{y}_0, \hat{y}_i) < v(\tilde{y}, \hat{y}_i)$ for $i = 1, 2$. Since $v(y, \hat{y}_i)$ is increasing in y if $y < \hat{y}_i$, we have $\hat{y}_0 < e(\hat{y}_i) \leq \tilde{y}$, and $e(\hat{y}_i)$ is the policy y that is closest to the chair's ideal that leaves legislator i indifferent between y and $\tilde{y}$. Note that $e(\hat{y}_i)$ is nondecreasing in $\hat{y}_i$, and, in particular, $e(\hat{y}_i) = \tilde{y}$ if $\hat{y}_i \geq \tilde{y}$ and $e(\hat{y}_i) < \hat{y}_i < \tilde{y}$ if $\hat{y}_i < \tilde{y}$. Figure 1 illustrates $e(\hat{y}_i)$ for $i = 1, 2$ when $\hat{y}_1 < \tilde{y} < \hat{y}_2$.

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and s, but this does not affect the chair's equilibrium behavior in the following sense. Consider an acceptance rule γ_i' that satisfies (E1) except that legislator i may reject some z when he is indifferent between z and s. Fix μ and a message profile m sent under μ. Suppose z' is a best response to m under (μ, γ') and $u_0(z') \geq u_0(s)$. (This is a reasonable assumption because if $u_0(z') < u_0(s)$, then z' is a best response only if the highest payoff the chair can achieve is $u_0(s)$ and she is sure that z' will be rejected.) We show that z' is still a best response to m if the legislators play (μ, γ), where γ satisfies (E1). Let $U_0^{\gamma}(z)$ be the chair's expected payoff from proposing z in response to m under (μ, γ). Define $U_0^{\gamma'}(z)$ analogously. Suppose to the contrary that z' is not a best response under (μ, γ). Then there exists z such that $U_0^{\gamma}(z) > U_0^{\gamma}(z') \geq U_0^{\gamma'}(z')$, where the last inequality holds since $u_0(z') \geq u_0(s)$ and z' is accepted with a higher probability under γ. Note that even under γ', the chair can achieve an expected payoff arbitrarily close to $U_0^{\gamma}(z)$ by raising the transfers to the legislators by an infinitesimal amount. Hence, there exists proposal z_ε such that $U_0^{\gamma'}(z_\varepsilon) > U_0^{\gamma'}(z')$, contradicting that z' is a best response under (μ, γ'). Thus, any optimal proposal under the alternate acceptance rule γ' is still optimal under the acceptance rule that satisfies (E1). In this sense, (E1) does not restrict the chair's equilibrium behavior.

[11]This is not a restrictive assumption if $c > 0$ because the chair strictly prefers the proposal $(\tilde{y}; c, 0, 0)$ (which is accepted with probability 1) to the status quo, so z is not a best response. If $c = 0$, however, it is possible that z is a best response, but not a unique one (for example, s is another best response).

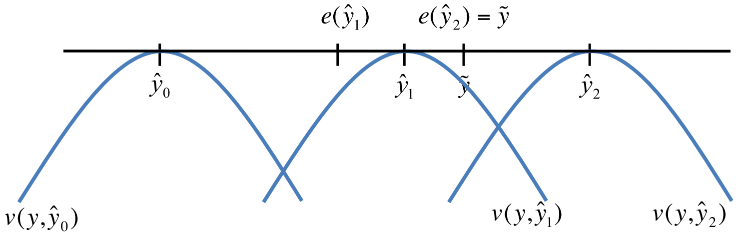

FIGURE 1. Illustration of $e(\hat{y}_i)$ for $i = 1, 2$ when $\hat{y}_1 < \tilde{y} < \hat{y}_2$.

To start, suppose the chair faces only legislator 1 who has veto power. This means that for any proposal to pass, legislator 1 must vote for it. Given θ_1, the chair chooses $z^1(\theta_1) = (y^1(\theta_1); x^1(\theta_1))$ to solve[12]

$$\max_{z \in Y \times X} u_0(z) = c - x_1 + \theta_0 v(y, \hat{y}_0)$$

subject to $x_1 + \theta_1 v(y, \hat{y}_1) \geq \theta_1 v(\tilde{y}, \hat{y}_1)$. Since $u_0(z)$ is decreasing in x_1, for x_1^1 to be optimal, it satisfies $x_1^1 = \theta_1(v(\tilde{y}, \hat{y}_1) - v(y^1, \hat{y}_1))$.[13] To satisfy $x_1^1 \geq 0$, we must have $v(\tilde{y}, \hat{y}_1) \geq v(y^1, \hat{y}_1)$. Thus, substituting for x_1 in the chair's maximization problem, y^1 must be a solution to

$$\max_{y \in Y} c - \theta_1(v(\tilde{y}, \hat{y}_1) - v(y, \hat{y}_1)) + \theta_0 v(y, \hat{y}_0)$$

subject to $v(\tilde{y}, \hat{y}_1) \geq v(y, \hat{y}_1)$. Since $v_{11} < 0$, the objective function is strictly concave and y^1 is unique. If $\theta_1 v_1(e(\hat{y}_1), \hat{y}_1) + \theta_0 v_1(e(\hat{y}_1), \hat{y}_0) \geq 0$, which holds when θ_1 is sufficiently high, then $v(\tilde{y}, \hat{y}_1) \geq v(y, \hat{y}_1)$ is binding, and we have a corner solution $y^1 = e(\hat{y}_1)$ and $x_1^1 = 0$. Otherwise, there exists a unique $y^1 < e(\hat{y}_1)$ such that $\theta_1 v_1(y^1, \hat{y}_1) + \theta_0 v_1(y^1, \hat{y}_0) = 0$ and $x_1^1 > 0$.[14] Let θ_1^{NT} satisfy $\theta_1^{\text{NT}} v_1(e(\hat{y}_1), \hat{y}_1) + \theta_0 v_1(e(\hat{y}_1), \hat{y}_0) = 0$. That is, θ_1^{NT} is the lowest θ_1 for which the chair's optimal proposal gives no transfer to legislator 1 ($x_1^1 = 0$) and has $y^1 = e(\hat{y}_1)$.

When the chair faces two legislators instead of one, her bargaining position is improved since the voting rule is the majority rule. Let $z^2(\theta_2)$ denote the chair's optimal proposal when she faces only legislator 2 with ideological intensity θ_2. If $u_0(z^i(\theta_i)) \geq u_0(z^j(\theta_j))$, then it is optimal for the chair to propose $(y; x)$ such that $y = y^i(\theta_i)$, $x_0 = x_0^i(\theta_i)$, $x_i = x_i^i(\theta_i)$, and $x_j = 0$ when she faces both legislators i and j. Notice that it is possible that the legislator whose ideal policy is further away from the chair's is included in an optimal proposal. This can happen when he puts sufficiently less weight on ideology than the other legislator does.

We now turn to the analysis of the model with incomplete information.

[12]For notational convenience, even when the chair faces only legislator $i \in \{1, 2\}$, we still assume that the chair's proposal z is in $Y \times X$, and let $x_j = 0$ for $j \in \{1, 2\}$ and $j \neq i$.

[13]To simplify notation, we suppress the dependence of y^1 and x^1 on θ_1.

[14]We show here that under complete information, the optimal proposal must involve $y \leq e(\hat{y}_1)$. Since this is true for an arbitrary θ_1, it follows that the optimal proposal must involve $y \leq e(\hat{y}_1)$ even under incomplete information.

4. One sender

Although our focus is on the game with three players and majority rule, it is useful to first consider a simpler game with one legislator (sender) other than the chair. In addition to providing useful intuition, the analysis is interesting in its own right because it is applicable to bilateral bargaining over two issues. Let Γ^S denote the game in which the set of legislators other than the chair is S. In this section, we consider the case with $S = \{1\}$.

The modification of the players' strategies and equilibrium conditions in $\Gamma^{\{1\}}$ are straightforward and are omitted. To classify equilibria, we define the *size* of an equilibrium to be the number of proposals elicited in that equilibrium. To characterize equilibria, we first establish the following lemma. (Proofs are given in the Appendix except for the proof of Lemma 4, which is given in the Supplementary Appendix available in a supplementary file on the journal website, http://econtheory.org/supp/821/supplement.pdf.)

Lemma 1. *(i) If type θ_1 weakly prefers $z' = (y'; x')$ to $z = (y; x)$, where $x'_1 > x_1$, then any type $\theta'_1 < \theta_1$ strictly prefers z' to z. (ii) If type θ_1 weakly prefers $z'' = (y''; x'')$ to $z = (y; x)$, where $x''_1 < x_1$, then any type $\theta''_1 > \theta_1$ strictly prefers z'' to z.*

A special case of Lemma 1 is worth noting: Suppose type θ_1 is indifferent between the status quo s and $z = (y; x)$, where $x_1 > 0$. If $\theta'_1 < \theta_1$, then type θ'_1 strictly prefers z to s; if $\theta'_1 > \theta_1$, then type θ'_1 strictly prefers s to z. This immediately implies that legislator 1 does not fully reveal his type in equilibrium.[15] To see this, note that in a separating equilibrium, legislator 1 receives only his status quo payoff as the chair would make a proposal that leaves him just willing to accept. But then type θ_1 would want to mimic a higher type by exaggerating his ideological intensity so as to get a better deal from the chair. In fact, we have a much stronger result that says that for any equilibrium, at most one proposal elicited in it gives legislator 1 some positive private benefit, and an equilibrium has at most size 2. But before deriving this result and characterizing size-2 equilibria, we first characterize size-1 equilibria.

4.1 *Size-1 equilibria*

We focus on babbling equilibrium, since any size-1 equilibrium is outcome equivalent to a babbling equilibrium. Let z' be the proposal elicited in a babbling equilibrium.

To find z', note that by Lemma 1, if $u_1(z, \overline{\theta}_1) \geq u_1(s, \overline{\theta}_1)$, then $u_1(z, \theta_1) \geq u_1(s, \theta_1)$ for all $\theta_1 \in \Theta_1$ and z is always accepted; if $u_1(z, \underline{\theta}_1) < u_1(s, \underline{\theta}_1)$, then $u_1(z, \theta_1) < u_1(s, \theta_1)$ for all $\theta_1 \in \Theta_1$ and z is always rejected; if $u_1(z, \overline{\theta}_1) < u_1(s, \overline{\theta}_1)$ and $u_1(z, \underline{\theta}_1) \geq u_1(s, \underline{\theta}_1)$, then there exists $\theta_1 \in \Theta_1$ such that $u_1(z, \theta_1) = u_1(s, \theta_1)$ and z is accepted with probability $F_1(\theta_1)$.

[15] To be more precise, legislator 1 does not fully reveal his type in equilibrium except in the degenerate case where $z^1(\theta_1) = (e(\hat{y}_1); c, 0, 0)$ for every $\theta_1 \in \Theta_1$. In this case, even if legislator 1 fully reveals his type, the chair still makes the same proposal $(e(\hat{y}_1); c, 0, 0)$ and we have a size-1 equilibrium.

Let $t_1(z)$ denote the highest type who is willing to accept z if z is accepted with positive probability and set $t_1(z)$ to $\underline{\theta}_1$ if z is accepted with probability 0. Formally

$$t_1(z) = \begin{cases} \max\{\theta_1 \in \Theta_1 : u_1(z, \theta_1) \geq u_1(s, \theta_1)\} & \text{if } u_1(z, \underline{\theta}_1) \geq u_1(s, \underline{\theta}_1) \\ \underline{\theta}_1 & \text{otherwise.} \end{cases}$$

For z' to be the proposal elicited in a babbling equilibrium, it must satisfy

$$z' \in \arg\max_{z \in Y \times X} u_0(z) F_1(t_1(z)) + u_0(s)\big[1 - F_1(t_1(z))\big].$$

We can also formulate the chair's problem as choosing the highest type who is willing to accept her proposal. Let $\theta_1' = t_1(z')$ and let $V(\theta_1) = u_0(z^1(\theta_1))$ denote the chair's highest payoff when facing legislator 1 of type θ_1. Then we have

$$\theta_1' \in \arg\max_{\theta_1 \in \Theta_1} V(\theta_1) F_1(\theta_1) + u_0(s)(1 - F_1(\theta_1)). \tag{1}$$

If the solution to (1) is unique, it is without loss of generality to consider only pure strategies. If the objective function is strictly concave, then θ_1' is unique. Another sufficient condition for uniqueness is that the objective function is strictly increasing in θ_1. Lemma A.1 in the Supplementary Appendix shows that in the uniform-quadratic case (i.e., θ_1 is uniformly distributed and $v(y, \hat{y}_1) = -(y - \hat{y}_1)^2$), (i) if $\hat{y}_1 < \tilde{y}$, then the objective function is strictly increasing in θ_1 and (1) has a unique solution at $\overline{\theta}_1$, and (ii) if $\hat{y}_1 \geq \tilde{y}$ and $c > 0$, then (1) may have an interior solution and a solution at $\overline{\theta}_1$, but this happens only nongenerically (specifically, fix all the parameters except for c; then there exists at most one value of c for which the solution to (1) is not unique).

4.2 *Size-2 equilibria*

The main finding in this subsection is that legislator 1 can credibly convey some information, but only in a limited way. We first show that the number of proposals elicited in an equilibrium is at most 2, and then we characterize size-2 equilibria and provide existence conditions.

The following lemma says that there is at most one proposal elicited in equilibrium that gives legislator 1 a strictly positive transfer.

Lemma 2. *Suppose proposals $z' = (y'; x')$ and $z'' = (y''; x'')$ are elicited in an equilibrium of $\Gamma^{\{1\}}$. If $x_1' > 0$ and $x_1'' > 0$, then $z' = z''$.*

To gain some intuition, suppose there are two equilibrium proposals z' and z'' that give legislator 1 positive transfers. Since the types who elicit z' and accept it weakly prefer it to s, and not all types who elicit z' strictly prefer z' to s (otherwise z' would not be optimal), there exists a type θ_1' who elicits z' and is indifferent between z' and s. Similarly, there exists a type θ_1'' who elicits z'' and is indifferent between z'' and s. Assume $\theta_1'' > \theta_1'$. Then by Lemma 1, type θ_1' strictly prefers to elicit z'' because he receives a payoff strictly higher than his status quo payoff by doing so, a contradiction. So only one equilibrium proposal can have $x_1 > 0$. Such a proposal must have $y < e(\hat{y}_1)$. When proposing it, the chair makes some transfer to legislator 1 in exchange for moving the policy toward her own ideal.

Now consider an equilibrium proposal $(y; x)$ with $x_1 = 0$. If $e(\hat{y}_1) \leq y \leq \tilde{y}$, all types accept it; if $y < e(\hat{y}_1)$, no type accepts it. Since $v(y, \hat{y}_0)$ is decreasing in y when $y \geq e(\hat{y}_1)$, we have $y = e(\hat{y}_1)$. Hence there are at most two proposals elicited in an equilibrium: one is $(e(\hat{y}_1); c, 0, 0)$ and the other is $(y; c - x_1, x_1, 0)$ with $y < e(\hat{y}_1)$ and $x_1 > 0$. Henceforth, denote the no-transfer proposal $(e(\hat{y}_1); c, 0, 0)$ by z^{NT}. Let θ_1^* be the type that is indifferent between $(y; c - x_1, x_1, 0)$ and z^{NT}. By Lemma 1, if $\theta_1 < \theta_1^*$, then type θ_1 strictly prefers $(y; c - x_1, x_1, 0)$ to z^{NT} and hence elicits $(y; c - x_1, x_1, 0)$. If $\theta_1 > \theta_1^*$, then type θ_1 strictly prefers z^{NT} to $(y; c - x_1, x_1, 0)$. A type $\theta_1 > \theta_1^*$ may elicit z^{NT} and accept it or may elicit $(y; c - x_1, x_1, 0)$ and reject it: either way he gets his status quo payoff. The following proposition provides a summary.

Proposition 1. *(i) In $\Gamma^{\{1\}}$, at most two proposals are elicited in any equilibrium. (ii) In a size-2 equilibrium in $\Gamma^{\{1\}}$ the elicited proposals are z^{NT} and $(y; c - x_1, x_1, 0)$ with $y < e(\hat{y}_1)$ and $x_1 > 0$. There exists a type θ_1^* such that if $\theta_1 < \theta_1^*$, type θ_1 elicits $(y; c - x_1, x_1, 0)$ and accepts it; if $\theta_1 \geq \theta_1^*$, type θ_1 either elicits $(y; c - x_1, x_1, 0)$ and rejects it or elicits z^{NT} and accepts it.*

Proposition 1 says that a type above θ_1^* may either elicit $(y; c - x_1, x_1, 0)$ and reject it or elicit z^{NT} and accept it. Note, however, that if there were any possibility of a "tremble" by legislator 1 at the voting stage, so that he might not carry out a planned veto and instead vote for a proposal even though he strictly prefers s to it, then his best message rule is to safely elicit z^{NT} if $\theta_1 > \theta_1^*$. The chair benefits if all $\theta_1 > \theta_1^*$ elicit z^{NT}, since she (weakly) prefers z^{NT} to s.

Suppose the types who elicit the same proposal in equilibrium send the same message,[16] and message m_1^c induces $(y; c - x_1, x_1, 0)$ and message m_1^f induces z^{NT}. We can interpret m_1^c as the "compromise" message and m_1^f as the "fight" message. When the chair receives m_1^c, she infers that legislator 1 is likely to have a low ideological intensity and responds with a compromise proposal that moves the policy toward her own ideal. When the chair receives m_1^f, she infers that legislator 1 is intensely ideological, and responds with a proposal that involves minimum policy change and no transfer for legislator 1. This proposal in response to the fight message passes with probability 1.[17] Note that multiple size-2 equilibria exist with a different set of elicited proposals that correspond to different thresholds θ_1^*.

Recall that $z^1(\theta_1)$ is the chair's optimal proposal when θ_1 is known.

Proposition 2. *A size-2 equilibrium exists in $\Gamma^{\{1\}}$ if and only if (i) $z^1(\overline{\theta}_1) = z^{\mathrm{NT}}$ and (ii) $z^1(\underline{\theta}_1) = (y; c - x_1, x_1, 0)$ for some $y < e(\hat{y}_1)$ and $x_1 > 0$.*

[16] This loses no generality because any size-2 equilibrium is outcome equivalent to such an equilibrium.

[17] Of course, the proposal induced by the fight message could be just maintaining the status quo. As such, the passage of such a proposal can be interpreted as inaction by the chair on policy change.

The conditions in Proposition 2 require the chair's optimal proposal to be z^{NT} when she is sure that legislator 1 is of the highest type, and to be a proposal that has $y < e(\hat{y}_1)$ and $x_1 > 0$ when she is sure that legislator 1 is of the lowest type. Intuitively, under these conditions, there exists a type $\theta_1^* \in \Theta_1$ such that z^{NT} is optimal when the chair believes that $\theta_1 \geq \theta_1^*$ and $(y; c - x_1, x_1, 0) \neq z^{\mathrm{NT}}$ is optimal when the chair believes that $\theta_1 \leq \theta_1^*$, which in turn guarantees that a size-2 equilibrium exists. From the analysis in Section 3, the existence conditions in Proposition 2 are satisfied if $\overline{\theta}_1 \geq \theta_1^{\mathrm{NT}} > \underline{\theta}_1$, where θ_1^{NT} is the lowest type of legislator 1 for which the chair's optimal proposal is z^{NT}.

4.3 *Comparative statics: Equilibria of different sizes*

A natural question is whether the players are better off in an equilibrium of a higher size. Clearly, the chair (weakly) prefers a size-2 equilibrium to a size-1 equilibrium because her decisions are based on better information in a size-2 equilibrium. As to legislator 1, consider the following two cases. (i) Suppose z^{NT} is elicited in a size-1 equilibrium. Then legislator 1's payoff is the same as his status quo payoff. Since in any size-2 equilibrium, the payoff of any type $\theta_1 \geq \theta_1^*$ is the same as his status quo payoff and the payoff of any type $\theta_1 < \theta_1^*$ is strictly higher than his status quo payoff, legislator 1 is better off in a size-2 equilibrium. (ii) Suppose $z' \neq z^{\mathrm{NT}}$ is elicited in a size-1 equilibrium. If z' is rejected with positive probability in the size-1 equilibrium, then a size-2 equilibrium exists in which every type of legislator 1 has the same payoff as in the size-1 equilibrium.[18] In this sense, legislator 1 is again (weakly) better off in a size-2 equilibrium.[19]

5. Two senders

We now analyze $\Gamma^{\{1,2\}}$, the game with two legislators. Without loss of generality, assume that $\hat{y}_1 \leq \hat{y}_2$, which implies that $e(\hat{y}_1) \leq e(\hat{y}_2)$. Since legislator 1's ideal point is closer to the chair's, we call legislator 1 the *closer* legislator and call legislator 2 the *more distant* legislator. We restrict attention to a class of equilibria called *connected equilibria*. An equilibrium (μ, γ, π) is connected if for any $\theta_i' \leq \theta_i''$ and $i = 1, 2$, $\mu_i(\theta_i') = \mu_i(\theta_i'')$ implies that $\mu_i(\theta_i) = \mu_i(\theta_i')$ for any $\theta_i \in [\theta_i', \theta_i'']$. In a connected equilibrium, the set of types who send the same message is an interval, possibly a singleton. We discuss a class of disconnected equilibria and why they are not robust to trembles at the voting stage toward the end of Section 5.2.

5.1 *Proposals elicited in connected equilibria*

Say that a proposal $(y; x)$ is a *one-transfer proposal* if either $x_1 > 0$ or $x_2 > 0$ but not both, is a *two-transfer proposal* if both $x_1 > 0$ and $x_2 > 0$, and is a *no-transfer proposal*

[18] To construct it, let $\theta_1' < \overline{\theta}_1$ be the type just willing to accept z'. Let $\mu_1(\theta_1) = m_1^c$ if $\theta_1 \leq \theta_1'$, $\mu_1(\theta_1) = m_1^f$ if $\theta_1 > \theta_1'$, $\pi(m_1^c) = z'$, $\pi(m_1^f) = z^{\mathrm{NT}}$, and $\pi(m) \in \{\pi(m_1^c), \pi(m_1^f)\}$ for any other $m_1 \in M_1$. In this size-2 equilibrium, the payoff for any $\theta_1 < \theta_1'$ is $u_1(z', \theta_1)$ and the payoff for any $\theta_1 \geq \theta_1'$ is $u_1(s, \theta_1)$, the same as in the size-1 equilibrium.

[19] If z' is accepted with probability 1 in the size-1 equilibrium, then types who are sufficiently high are better off in the size-1 equilibrium than in any size-2 equilibrium since they receive payoffs higher than the status quo payoff in the size-1 equilibrium but just status quo payoff in the size-2 equilibrium.

if $x_1 = 0$ and $x_2 = 0$. The following lemma provides sufficient conditions under which no proposal elicited in a connected equilibrium is a two-transfer proposal. Suppose F_i has a differentiable density function f_i for $i = 1, 2$. Recall that $f_i(\theta_i)/(1 - F_i(\theta_i))$ is the hazard rate and that F_i satisfies the strict increasing hazard rate property (IHRP) if $f_i(\theta_i)/(1 - F_i(\theta_i))$ is strictly increasing in θ_i.

Lemma 3. *If the prior F_i $(i = 1, 2)$ satisfies IHRP in $\Gamma^{\{1,2\}}$, then any proposal elicited in a connected equilibrium has $x_i > 0$ for at most one $i \neq 0$.*

To see why Lemma 3 holds, consider the support of the chair's posterior. Suppose it is a singleton for at least one of the legislators, say legislator 1. Then, given any proposal, the chair knows whether legislator 1 will accept or reject it. A two-transfer proposal is not optimal, because if legislator 1 accepts it, then the chair is strictly better off reducing x_2, and if legislator 1 rejects it, then the chair is strictly better off reducing x_1. Now suppose the support of the posterior on θ_i is not a singleton for both $i = 1, 2$. Then, for a fixed sum of the transfers to legislators 1 and 2, an optimal proposal minimizes the probability that the proposal is rejected. Under IHRP, the rejection probability is strictly concave in the transfer to each legislator, implying that the solution cannot be interior. Hence, a two-transfer proposal is again not optimal. Many commonly used distribution functions, including uniform, normal, log-normal, and beta distributions, satisfy IHRP. This property is also frequently assumed in economics and political science applications.[20]

The next two lemmas establish some properties of no-transfer proposals and one-transfer proposals, which are useful in equilibrium characterization.

Lemma 4. *Suppose $z = (y; x)$ is elicited in an equilibrium of $\Gamma^{\{1,2\}}$ with $x_1 = x_2 = 0$. Then (i) $y = e(\hat{y}_1)$, (ii) $u_1(z, \theta_1) = u_1(s, \theta_1)$ for any $\theta_1 \in \Theta_1$, (iii) if $e(\hat{y}_1) = e(\hat{y}_2)$, then $u_2(z, \theta_2) = u_2(s, \theta_2)$ for any $\theta_2 \in \Theta_2$, and (iv) if $e(\hat{y}_1) < e(\hat{y}_2)$, then $u_2(z, \theta_2) < u_2(s, \theta_2)$ for any $\theta_2 \in \Theta_2$; as such, legislator 2 rejects z and legislator 1 is pivotal with respect to z.*

To see why Lemma 4 holds, suppose z is elicited in an equilibrium with $x_1 = x_2 = 0$. Since $e(\hat{y}_1) \le \hat{y}_1 \le \hat{y}_2$, if $y < e(\hat{y}_1)$, neither legislator accepts z, and if $e(\hat{y}_1) \le y \le \tilde{y}$, at least legislator 1 accepts z. Since $v(y, \hat{y}_0)$ is decreasing in y when $y \ge e(\hat{y}_1) > \hat{y}_0$, it is optimal to propose $y = e(\hat{y}_1)$. So the optimal no-transfer proposal z is equal to $(e(\hat{y}_1); c, 0, 0)$, denoted by z^{NT}. Since $x_1 = 0$ and $y = e(\hat{y}_1)$, we have $u_1(z, \theta_1) = u_1(s, \theta_1)$ for any θ_1. Similarly, if $e(\hat{y}_1) = e(\hat{y}_2)$, we have $u_2(z, \theta_2) = u_2(s, \theta_2)$ for any θ_2. Since $v(y, \hat{y}_2)$ is increasing in y when $y < \hat{y}_2$, if $e(\hat{y}_1) < e(\hat{y}_2) \le \hat{y}_2$, we have $u_2(z, \theta_2) < u_2(s, \theta_2)$ for any θ_2. It follows that legislator 2 rejects z and legislator 1 is pivotal. The next lemma says that the legislator who is excluded in a one-transfer proposal rejects it, making the legislator who is included pivotal.

Lemma 5. *Suppose $z = (y; x)$ is elicited in an equilibrium in $\Gamma^{\{1,2\}}$ and $x_i > 0$, $x_j = 0$. Then $u_j(s, \theta_j) > u_i(z, \theta_j)$ for all $\theta_j \in \Theta_j$ and $u_i(z, \theta_i) \ge u_i(s, \theta_i)$ for some $\theta_i \in \Theta_i$. Hence legislator j rejects z and legislator i is pivotal with respect to z.*

[20]See Bagnoli and Bergstrom (2005) for a list of distribution functions that satisfy the increasing hazard rate property and references to some of the seminal papers that assume it.

If F_1 and F_2 satisfy IHRP, then by Lemma 3, any proposal elicited in a connected equilibrium is either a no-transfer or a one-transfer proposal. This simplifies the problem of characterizing elicited proposals in a connected equilibrium. Specifically, recall that $t_i(z)$ is the highest type willing to accept z if some θ_i weakly prefers z to s, and $t_i(z) = \underline{\theta}_i$ otherwise. Suppose the chair's posterior is $G = (G_1, G_2)$. Let $\beta(z) = 1 - [1 - G_1(t_1(z))][1 - G_2(t_2(z))]$ and

$$z(G) \in \arg\max_{z \in Y \times X} u_0(z)\beta(z) + u_0(s)(1 - \beta(z)).$$

That is, $z(G)$ is an optimal proposal for the chair under belief G. Let $U_0(G)$ be the associated value function, that is, $U_0(G)$ is the highest expected payoff for the chair under belief G.

Similarly, let $z^{-j}(G_i)$ be a proposal that gives the chair the highest expected payoff among all the proposals that exclude legislator j, under belief G_i ($i \neq j$), and let $U_0^{-j}(G_i)$ be the associated value function. Note that $z^{-j}(G_i)$ does not depend on G_j, because if a proposal excludes j, either every type of legislator j accepts it or no type of legislator j accepts it.

Suppose F_1 and F_2 satisfy IHRP. Fix a connected equilibrium (μ, γ, π). Let $H(m) = (H_1(m_1), H_2(m_2))$ be the chair's posterior when receiving m. By Lemma 3, for any m sent in this equilibrium, $z(H(m))$ is not a two-transfer proposal and, therefore, $U_0(H(m)) = \max_{i=1,2} U_0^{-j}(H_i(m_i))$. Note that $U_0^{-j}(H_i(m_i)) \geq u_0(z^{\mathrm{NT}})$ for $i = 1, 2$. Thus, if $U_0^{-j}(H_i(m_i)) > U_0^{-i}(H_j(m_j))$, then it is optimal for the chair to exclude j and include i. If $U_0^{-j}(H_i(m_i)) = u_0(z^{\mathrm{NT}})$ for $i = 1, 2$, then z^{NT} is an optimal proposal for the chair.

Since a babbling equilibrium is a connected equilibrium, all the results established for connected equilibria apply to babbling equilibria. Specifically, suppose F_1 and F_2 satisfy IHRP. If $U_0^{-j}(F_i) > U_0^{-i}(F_j) \geq u_0(z^{\mathrm{NT}})$, then the proposal elicited in any babbling equilibrium includes i and excludes j; if $U_0^{-2}(F_1) = U_0^{-1}(F_2) > u_0(z^{\mathrm{NT}})$, then the proposal elicited in any babbling equilibrium is a one-transfer proposal that includes either 1 or 2; if $U_0^{-2}(F_1) = U_0^{-1}(F_2) = u_0(z^{\mathrm{NT}})$, then there exists a babbling equilibrium in which the no-transfer proposal z^{NT} is elicited.

5.2 *Informative equilibria*

In this section, we characterize equilibria in $\Gamma^{\{1,2\}}$ in which some information is transmitted. Throughout this section, we assume that F_1 and F_2 satisfy IHRP.

Fix a connected equilibrium (μ, γ, π), and consider the proposals $\pi(m)$ and $\pi(m')$, where $m_i = m'_i$ for some $i \in \{1, 2\}$. Suppose both $\pi(m)$ and $\pi(m')$ exclude legislator $j \neq i$. That is, $z^{-j}(H_i(m_i))$ is an optimal proposal when the chair receives m and $z^{-j}(H_i(m'_i))$ is an optimal proposal when she receives m'. If $z^{-j}(H_i(m_i))$ is unique, then, since $m_i = m'_i$, and both $\pi(m)$ and $\pi(m')$ exclude j, we must have $\pi(m) = \pi(m') = z^{-j}(H_i(m_i))$. If $z^{-j}(H_i(m_i))$ is not unique, then conceivably $\pi(m) \neq \pi(m')$, but this requires that the chair chooses different proposals—none of which includes legislator j—for different messages sent by legislator j, although she has the same belief about legislator i.

We now consider a refinement that rules out the preceding scenario. Call a connected equilibrium (μ, γ, π) a *simple connected equilibrium* (SCE) if the following condition is satisfied: for any m and m' such that $m_i = m'_i$ for some $i \in \{1, 2\}$, if both $\pi(m)$ and $\pi(m')$ exclude legislator $j \neq i$, then $\pi(m) = \pi(m')$. We find this to be a reasonable refinement because when the chair optimally excludes legislator j, her proposal depends only on her belief about legislator i's type, which has nothing to do with what legislator j says. This refinement is also automatically satisfied if $z^{-j}(H_i(m_i))$ is unique. (Uniqueness of $z^{-j}(H_i(m_i))$ holds under some familiar functional forms: Lemma A.1 in the Supplementary Appendix shows that if $H_i(m_i)$ is a uniform distribution and $v(y, \hat{y}_i) = -(y - \hat{y}_i)^2$, then $z^{-j}(H_i(m_i))$ is unique.)

Let P_i denote the probability measure of the random variable θ_i that corresponds to F_i. Say that μ_i is a *size-1 message rule* if $\mu_i(\theta_i) = \mu_i(\theta'_i)$ for all $\theta_i, \theta'_i \in \Theta_i$ and that μ_i is a *size-2 message rule* if there exists a set $A_i \subset \Theta_i$ with $P_i(\theta_i \in A_i) \in (0, 1)$ such that (i) $\mu_i(\theta_i) = \mu_i(\theta'_i)$ if either $\theta_i, \theta'_i \in A_i$ or $\theta_i, \theta'_i \in \Theta_i \setminus A_i$ and (ii) $\mu_i(\theta_i) \neq \mu_i(\theta'_i)$ if $\theta_i \in A_i$ and $\theta'_i \in \Theta_i \setminus A_i$.

Fix an equilibrium (μ, γ, π). Say that μ_i is *equivalent to* μ'_i if for almost all $(\theta_1, \theta_2) \in \Theta_1 \times \Theta_2$, we have $\pi(\mu_i(\theta_i), \mu_j(\theta_j)) = \pi(\mu'_i(\theta_i), \mu_j(\theta_j))$.[21] The message rule μ_i is equivalent to μ'_i in the sense that the joint distributions on type profiles and proposals are the same under μ_i and μ'_i, holding the other strategies in (μ, γ, π) fixed.

We say that legislator i is *uninformative* in equilibrium (μ, γ, π) if $\pi(\mu_i(\theta'_i), \mu_j(\theta_j)) = \pi(\mu_i(\theta_i), \mu_j(\theta_j))$ for almost all $(\theta_i, \theta'_i, \theta_j) \in \Theta_i \times \Theta_i \times \Theta_j$ and that legislator i is *informative* in (μ, γ, π) otherwise.[22] The condition for legislator i being uninformative in an equilibrium is the same as requiring that his message rule is equivalent to a size-1 message rule. Say that (μ, γ, π) is an *informative equilibrium* if at least one legislator is informative in it.

For any $z \in Y \times X$, let $I_i(z) = 1$ if z includes legislator i and let $I_i(z) = 0$ if z excludes legislator i. Let $q_i(m_i) = \int_{\Theta_j} I_i(\pi(m_i, \mu_j(\theta_j)))\, dF_j$ be the probability that legislator i is included when sending m_i in (μ, γ, π).

Proposition 3. *Suppose F_1 and F_2 satisfy IHRP. Fix a simple connected equilibrium (μ, γ, π) in $\Gamma^{\{1,2\}}$. Suppose legislator i is informative in this equilibrium. Then there exist $m_i^c, m_i^f \in M_i$ such that $q_i(m_i^c) > 0$ and $q_i(m_i^f) = 0$. Moreover, μ_i is equivalent to a size-2 message rule μ_i^{II} with the property that there exists $\theta_i^* \in (\underline{\theta}_i, \overline{\theta}_i)$ such that $\mu_i^{\mathrm{II}}(\theta_i) = m_i^c$ for $\theta_i < \theta_i^*$ and $\mu_i^{\mathrm{II}}(\theta_i) = m_i^f$ for $\theta_i > \theta_i^*$.*

[21] To simplify notation, we use $(\mu_i(\theta_i), \mu_j(\theta_j))$ to denote a message profile in which legislator i sends $\mu_i(\theta_i)$ and legislator j sends $\mu_j(\theta_j)$. We use analogous notation for other vectors of variables that involve legislators i and j.

[22] It is possible that legislator i partially reveals his type in (μ, γ, π), but he is still uninformative by our definition. For example, suppose the chair optimally excludes i for θ_i sufficiently close to $\overline{\theta}_i$. Suppose also that $\mu_i(\theta_i)$ reveals θ_i if $\theta_i \in (\overline{\theta}_i - \varepsilon, \overline{\theta}_i]$ for ε sufficiently small. Although the chair updates her belief for messages sent by $\theta_i \in (\overline{\theta}_i - \varepsilon, \overline{\theta}_i]$, her proposal does not depend on $\mu_i(\theta_i)$. Since the information about θ_i is useless for the chair's decision and irrelevant for the outcome, we consider legislator i to be uninformative in this case.

Proposition 3 says that in any SCE, legislator i can convey only a limited amount of information in that even when informative, his message rule is equivalent to a size-2 message rule. To give a sketch of the proof, we first show that in (μ, γ, π), there exists at most one m_i sent by a positive measure of θ_i such that $q_i(m_i) > 0$; when such a message exists, the types who send this message form an interval at the lower end of Θ_i. We also show that there exists at most one message m_i sent by a single type such that $q_i(m_i) > 0$. So at most two m_i's have the property that $q_i(m_i) > 0$ and one of them is sent by only a single type. Consider the following two possibilities: (a) Suppose there exists no m_i sent with positive probability such that $q_i(m_i) > 0$. Then $q_i(\mu_i(\theta_i)) = 0$ for almost all $\theta_i \in \Theta_i$. Since the proposal and the resulting outcome do not depend on m_i if legislator i is excluded in an SCE, it follows that μ_i is equivalent to a size-1 message rule such that every θ_i sends the same message that results in zero probability that legislator i is included. (b) Suppose m_i^c is sent with positive probability and $q_i(m_i^c) > 0$. If legislator i is informative, then there exists m_i^f (sent by some type) such that $q_i(m_i^f) = 0$. Since in (μ, γ, π), the types who send m_i^c form an interval at the lower end of Θ_i, there exists a threshold θ_i^* such that any type below θ_i^* sends m_i^c and almost every type above θ_i^* sends a message that results in zero probability that i is included. Hence μ_i is equivalent to μ_i^{II} such that $\mu_i^{\mathrm{II}}(\theta_i) = m_i^c$ for $\theta_i < \theta_i^*$ and $\mu_i^{\mathrm{II}}(\theta_i) = m_i^f$ for $\theta_i > \theta_i^*$.

The next proposition says that at most one legislator is informative in an SCE.

Proposition 4. *Suppose F_1 and F_2 satisfy IHRP. Fix a simple connected equilibrium (μ, γ, π) in $\Gamma^{\{1,2\}}$. (i) At most one legislator is informative in (μ, γ, π). (ii) If $e(\hat{y}_1) < e(\hat{y}_2)$, then legislator 2 is uninformative in (μ, γ, π).*

To gain some intuition for Proposition 4, imagine that both legislators are informative in (μ, γ, π). Then, by Proposition 3, both legislators are included with positive probability. By Lemmas 3 and 5, a legislator's payoff is weakly higher than his status quo payoff when included, but strictly lower than his status quo payoff when the other legislator is included. So, independent of his type, each legislator has an incentive to send the message that generates the highest probability of inclusion. But as Proposition 3 shows, if a legislator is informative, then with positive probability, he sends a message that results in zero probability of inclusion, a contradiction. As to why legislator 2 is uninformative when $e(\hat{y}_1) < e(\hat{y}_2)$, note that in this case, under the no-transfer proposal z^{NT}, legislator 2's payoff is *strictly* lower than his status quo payoff. Therefore, between m_2^c and m_2^f as described in Proposition 3, every type of legislator 2 strictly prefers to send m_2^c (with $q_2(m_2^c) > 0$) than m_2^f (with $q_2(m_2^f) = 0$), again a contradiction.

What are the proposals elicited in an informative equilibrium? Consider an SCE (μ, γ, π) in which legislator i is informative. For simplicity, assume $\mu_j(\theta_j) = m_j^*$ for all $\theta_j \in \Theta_j$, and $\mu_i(\theta_i) = m_i^c$ for $\theta_i < \theta_i^*$ and $\mu_i(\theta_i) = m_i^f$ for $\theta_i > \theta_i^*$, where $q_i(m_i^c) > 0$ and $q_i(m_i^f) = 0$. Since $q_i(m_i^f) = 0$, the proposal $\pi(m_i^f, m_j^*)$ excludes legislator i. Suppose $\pi(m_i^f, m_j^*)$ includes legislator j. Then, by Lemmas 3 and 5, legislator j is pivotal with respect to $\pi(m_i^f, m_j^*)$ and accepts it with positive probability, implying that by sending m_i^f, type θ_i's payoff is strictly lower than $u_i(s, \theta_i)$. Since $q_i(m_i^c) > 0$, the proposal $\pi(m_i^c, m_j^*)$

includes legislator i, and by Lemma 5, type θ_i's payoff by sending m_i^c is weakly higher than $u_i(s, \theta_i)$. Therefore, any type $\theta_i > \theta_i^*$ has an incentive to deviate and send m_i^c, a contradiction. It follows that $\pi(m_i^f, m_j^*)$ excludes j as well as i and $\pi(m_i^f, m_j^*) = z^{\text{NT}}$. Since $\pi(m_i^c, m_j^*)$ includes i, it must have $y < e(\hat{y}_1)$, $x_i > 0$, and $x_j = 0$. These arguments prove the next result.

Proposition 5. *Suppose F_1 and F_2 satisfy IHRP. Fix an informative simple connected equilibrium (μ, γ, π) in $\Gamma^{\{1,2\}}$ in which legislator i uses a size-2 message rule and legislator j uses a size-1 message rule. Then there exists $\theta_i^* \in (\underline{\theta}_i, \overline{\theta}_i)$ such that for any $\theta_j \in \Theta_j$, if $\theta_i > \theta_i^*$, then $\pi(\mu_i(\theta_i), \mu_j(\theta_j)) = z^{\text{NT}}$, and if $\theta_i < \theta_i^*$, then $\pi(\mu_i(\theta_i), \mu_j(\theta_j)) = (y; x)$ with $y < e(\hat{y}_1)$, $x_i > 0$, and $x_j = 0$.*

Similar to the one-sender case, we can interpret the message sent by types below θ_i^* as the compromise message and interpret the message sent by types above θ_i^* as the fight message. The chair responds to the compromise message with a proposal that gives legislator i some private benefit and moves the policy toward her own ideal, and responds to the fight message with a proposal that involves minimum policy change and gives no private benefit to either legislator. In Proposition 3, we showed that if an informative legislator sends the compromise message, then his probability of being included is positive. But as shown in Proposition 5, the result is stronger: the informative legislator is included with probability 1 when sending the compromise message.

To illustrate what an informative equilibrium looks like, we provide the following example.

Example 1. Suppose $\tilde{y} = 0$, $\hat{y}_0 = -1$, $\hat{y}_1 = -0.2$, $\hat{y}_2 = 0.5$, and $c = 1$. Assume $\theta_0 = 1$, θ_1 and θ_2 are both uniformly distributed on $[\frac{1}{4}, 4]$, and player i's utility function is $x_i - \theta_i(y - \hat{y}_i)^2$. ◊

Suppose $\mu_1(\theta_1) = m_1^c$ if $\theta_1 \in [\frac{1}{4}, 1]$, $\mu_1(\theta_1) = m_1^f$ if $\theta_1 \in (1, 4]$, and $\mu_2(\theta_2) = m_2^*$ for all θ_2.[23] Given the message rules, when the chair receives m_1^c, she infers that $\theta_1 \in [\frac{1}{4}, 1]$. Calculation shows that $\pi(m_1^c, m_2^*) = (-0.6; 0.88, 0.12, 0)$, a proposal that gives legislator 1 a positive transfer and moves the policy toward the chair's ideal.[24] When the chair receives m_1^f, she infers that $\theta_1 \in (1, 4]$. Calculation shows that it is optimal to propose $z^{\text{NT}} = (-0.4; 1, 0, 0)$. Intuitively, it is too costly for the chair to move the policy closer to her ideal because legislator 1 is too intensely ideological and legislator 2 holds an ideological position that is too far away.

[23]Here we let $\theta_1^* = 1$, but there are many other equilibria given by different thresholds.

[24]In this example, the proposal that the chair makes in response to (m_1^c, m_2^*) is accepted with probability 1 by legislator 1. There are examples in which a proposal made in response to a compromise message may fail to pass with positive probability. For example, suppose the distribution of θ_1 is a truncated exponential distribution on $[\frac{1}{4}, 4]$ with the parameter $\lambda = 4$, which implies that $F_1(\theta_1) = (e^{-1} - e^{-4x})/(e^{-1} - e^{-16})$. Keep all the other parametric assumptions unchanged and assume $\mu_1(\theta_1) = m_1^c$ if $\theta_1 \in [\frac{1}{4}, 2]$ and $\mu_1(\theta_1) = m_1^f$ if $\theta_1 \in (2, 4]$, and $\mu_2(\theta_2) = m_2^*$ for all θ_2. Then $\pi(m_1^c, m_2^*) = (-0.585; 0.883, 0.117, 0)$ and it is rejected by all types of legislator 2 and accepted by legislator 1 if and only if $\theta_1 \leq 1.076$. Hence it fails to pass with strictly positive probability.

Our analysis has focused on connected equilibria. Similar to $\Gamma^{\{1\}}$, disconnected equilibria may exist in $\Gamma^{\{1,2\}}$ in which types $\theta_i < \theta_i^*$ of legislator i elicit $(y; x)$ with $y < e(\hat{y}_1)$, $x_i > 0$, and $x_j = 0$, some types $\theta_i > \theta_i^*$ elicit z^{NT} and accept it, and other types $\theta_i > \theta_i^*$ elicit the same proposal as that elicited by types below θ_i^* and reject it, and legislator j babbles. (This class of disconnected equilibria is the only one we have found.) Note that similar to $\Gamma^{\{1\}}$, these disconnected equilibria are not robust to trembles by either legislator at the voting stage. That is, if either legislator might not carry out a planned rejection, then legislator i's best message rule is to safely elicit z^{NT} when $\theta_i > \theta_i^*$.[25]

We next provide conditions for the existence of an SCE in which legislator 1 is informative. (The conditions are similar for SCE in which legislator 2 is informative, with the additional requirement that $e(\hat{y}_1) = e(\hat{y}_2)$.)

The existence conditions for informative equilibria in $\Gamma^{\{1,2\}}$ are analogous to the existence conditions for size-2 equilibria in $\Gamma^{\{1\}}$, but with a modified condition (i) and with the additional requirement that it is optimal for the chair to exclude legislator 2, which is guaranteed by condition (iii).

Recall that θ_1^{NT} is the lowest type of legislator 1 for which the chair's optimal proposal is z^{NT}. Condition (i) in Proposition 2 says that z^{NT} is an optimal proposal when the chair is certain that legislator 1's type is $\overline{\theta}_1$, which is equivalent to $\overline{\theta}_1 \geq \theta_1^{\mathrm{NT}}$. Condition (i) in Proposition 6 strengthens this by requiring that $\overline{\theta}_1 > \theta_1^{\mathrm{NT}}$. We need this modification to rule out an equilibrium in which two different proposals are elicited, but one of them (z^{NT}) is elicited by a single type ($\overline{\theta}_1$) and, therefore, does not satisfy our definition of an informative equilibrium.

To see why condition (iii) guarantees that it is optimal for the chair to exclude legislator 2, recall that $U_0^{-1}(F_2)$ is the highest payoff the chair gets by excluding 1. If $U_0^{-1}(F_2) = u_0(z^{\mathrm{NT}})$, then no proposal that includes 2 gives the chair a higher payoff than z^{NT} and, therefore, it is optimal for the chair to exclude 2. Recall that $z^1(\theta_1)$ is the chair's optimal proposal when facing only legislator 1 with known θ_1. We have the following result (the proof is omitted since it is similar to that of Proposition 2).

Proposition 6. *Suppose F_1 and F_2 satisfy the IHRP. A simple connected equilibrium in which legislator 1 is informative exists if and only if (i) $\overline{\theta}_1 > \theta_1^{\mathrm{NT}}$, (ii) $z^1(\underline{\theta}_1) = (y; c - x_1, x_1, 0)$ for some $y < e(\hat{y}_1)$ and $x_1 > 0$, and (iii) $U_0^{-1}(F_2) = u_0(z^{\mathrm{NT}})$.*

Similar to $\Gamma^{\{1\}}$, condition (i) is satisfied if $\overline{\theta}_1$ is sufficiently high and condition (ii) is satisfied if $\underline{\theta}_1$ is sufficiently low. Condition (iii) is satisfied, roughly, if θ_2 is sufficiently likely to be high.

[25] These trembles at the voting stage imply that a legislator should elicit the proposal he prefers most. (With two senders, a legislator elicits a *distribution* of proposals since the chair's strategy is a function of both legislators' messages. When legislator j is uninformative, the distribution of proposals elicited by legislator i is degenerate and, in this case, trembles at the voting stage imply that he should elicit only the proposal he prefers most.) In addition to disconnected equilibria, some connected equilibria may not be robust to such trembles. For instance, in the example in footnote 24, some types below θ_1^* reject the proposal they elicit and they are better off by safely eliciting z^{NT} if there is any possibility of trembles at the voting stage. If the threshold θ_1^* is sufficiently low, however, then we have a connected equilibrium in which the proposal made in response to the compromise message is accepted with probability 1. Such a connected equilibrium is robust to trembles at the voting stage.

5.3 *Comparative statics*

Two comparisons seem especially interesting. The first is the comparison between informative and uninformative equilibria in $\Gamma^{\{1,2\}}$. The second is the comparison of equilibria in $\Gamma^{\{1,2\}}$ and those in $\Gamma^{\{1\}}$, which allows us to answer, "Is the chair always better off bargaining with more legislators?" Surprisingly, we show below that although the chair needs only one legislator's support to pass a proposal, she may be worse off when facing two legislators than just one.

Comparing informative and uninformative equilibria. Let E^u be an uninformative equilibrium and let E^{I} be an SCE in which legislator i is informative of $\Gamma^{\{1,2\}}$. Since the chair benefits from information transmission, she is better off in E^{I} than in E^u. The welfare comparison for the informative legislator is similar to that in the one-sender case (Section 4.3); in particular, he benefits from information transmission as well. The uninformative legislator j, however, may be worse off when legislator i is informative. To illustrate, suppose the proposal elicited in E^u is z^{NT}. Since in an informative SCE the elicited proposals are z^{NT} and $(y; x)$ with $y < e(\hat{y}_1)$ and $x_j = 0$, and legislator j strictly prefers $e(\hat{y}_1)$ to any $y < e(\hat{y}_1)$, he is better off in E^u.

Does it benefit the chair to face more legislators? Under complete information, the chair is clearly better off bargaining with two legislators than only one because she gains flexibility as to whom to make a deal with, as shown at the end of Section 3. Under asymmetric information, however, the answer is less clear. As illustrated in the following example, having two legislators may result in less information transmitted in equilibrium and this hurts the chair.

Example 2. Suppose $c = 1$, $\tilde{y} = 0$, $u_0(z) = x_0 - \theta_0(y - \hat{y}_0)^2$, where $\theta_0 = 1$, $\hat{y}_0 = -1$, and $u_1(z, \theta_1) = x_1 - \theta_1(y - \hat{y}_1)^2$, where $\hat{y}_1 = -0.2$ and θ_1 is uniformly distributed on $[\frac{1}{4}, 4]$. $\Diamond$

Consider $\Gamma^{\{1\}}$, in which the chair faces only legislator 1. Size-2 equilibria exists in $\Gamma^{\{1\}}$. For instance, analogous to Example 1, a size-2 equilibrium exists in which $\mu_1(\theta_1) = m_1^c$ if $\theta_1 \in [\frac{1}{4}, 1]$ and $\mu_1(\theta_1) = m_1^f$ if $\theta_1 \in (1, 4]$. The chair's payoff in this equilibrium is 0.656. Now consider $\Gamma^{\{1,2\}}$ in which the chair faces both legislators 1 and 2.[26] Suppose $u_2(z, \theta_2) = x_2 - \theta_2(y - \hat{y}_2)^2$, where $\hat{y}_2 = -0.201$ and θ_2 is uniformly distributed on $[5, 10]$. Since $e(\hat{y}_2) < e(\hat{y}_1)$, by Proposition 4(ii) (adapted to the case with $e(\hat{y}_2) < e(\hat{y}_1)$), legislator 1 is not informative in any SCE in $\Gamma^{\{1,2\}}$. Calculation shows that $z^2(\underline{\theta}_2) = (y; x)$, where $y = e(\hat{y}_2)$ and $x_2 = 0$. Since a necessary condition for the existence of an SCE in which legislator 2 is informative is $z^2(\underline{\theta}_2) = (y; x)$, where $y < e(\hat{y}_2)$ and $x_2 > 0$, legislator 2 is not informative in any SCE either. In any uninformative equilibrium of $\Gamma^{\{1,2\}}$, the proposal $(-0.402; 1, 0, 0)$ is elicited with probability 1 and the chair's payoff is 0.642, which is lower than 0.656, her payoff in the size-2 equilibrium that we identified in $\Gamma^{\{1\}}$.

In the preceding example, the chair is worse off when we add legislator 2 whose position is closer to the chair's (making it impossible for legislator 1 to be informative) but who is intensely ideological (making it impossible for himself to be informative). What

[26]Although earlier we assumed that $\hat{y}_1 \leq \hat{y}_2$ for expositional convenience, in this example, so as to discuss all possibilities, we allow $\hat{y}_1 > \hat{y}_2$.

happens if we add a legislator whose position is further away from the chair's? Can it still result in informational loss? The next example shows that the answer is yes. Suppose $\hat{y}_2 = -0.1$ and θ_2 is uniformly distributed on $[\frac{1}{4}, \frac{4}{5}]$. Since $e(\hat{y}_1) < e(\hat{y}_2)$, by Proposition 4, legislator 2 is uninformative in any SCE of $\Gamma^{\{1,2\}}$. Calculation shows that $z^{-1}(F_2) = (y; x)$, where $y = -0.6$ and $x_2 = 0.192$. This means that conditional on excluding legislator 1, the chair's proposal includes legislator 2. So condition (iii) in Proposition 6 fails and it is not possible for legislator 1 to be informative in any SCE of $\Gamma^{\{1,2\}}$ either.[27] In any uninformative equilibrium of $\Gamma^{\{1,2\}}$, the proposal $z^{-1}(F_2)$ is elicited with probability 1, resulting in a payoff of 0.648 for the chair, still lower than 0.656. So, the chair is again worse off when she faces two legislators rather than one.

To summarize, the chair may be better off bargaining with only one legislator when the informational loss resulting from having two legislators is sufficiently high. This contrasts with Krishna and Morgan (2001b), in which a decision maker is never worse off when facing two senders rather than one. In their model, the senders have the same information and for any equilibrium in the one-sender case, there exists an equilibrium when another sender is added that gives the decision maker a payoff at least as high as his original equilibrium payoff.

5.4 *Benefits of bundled bargaining*

In the model considered so far, the chair makes a proposal on an ideological dimension and a distributive dimension, and the two dimensions are accepted or rejected together. (Call this the *bundled bargaining* game.) A natural question is whether the chair is better off bundling the two dimensions or negotiating them separately. Specifically, consider a "separate bargaining" game in which the chair, after receiving the messages, makes a proposal on only the ideological dimension and another on only the distributive dimension. The legislators vote on each proposal separately. In this game, it is possible that a proposal on one dimension passes, while the proposal on the other dimension fails to pass.

The chair is better off in the bundled bargaining game. To see why, note that in the separate bargaining game, the legislators' private information is irrelevant since it is about how they trade off one dimension for the other, not about their preferences on either dimension. The resulting unique equilibrium outcome is z^{NT}. In the bundled bargaining game, z^{NT} is still feasible and will pass if proposed, and this immediately implies that the chair cannot be worse off. Indeed, bundling gives the chair two advantages: (i) useful information may be revealed in equilibrium, as seen in Proposition 5; (ii) given the information she has, the chair can use private benefit as an instrument to make better proposals that exploit the difference in how the players trade off the two dimensions. Because of these advantages, if the chair could choose between bundled bargaining and separate bargaining, she would choose the former. Legislator 1 gets his

[27] Intuitively, if legislator 1 is informative in some SCE, then the chair responds to his fight message by including legislator 2, making legislator 1 strictly worse off than the status quo and giving him an incentive to deviate.

status quo payoff and legislator 2 is worse off than the status quo in the separate bargaining game, but in the bundled bargaining game, the informative legislator is better off than the status quo, whereas the other (uninformative) legislator is worse off than the status quo. This result that the chair is better off in the bundled bargaining game is reminiscent of the finding in Jackson and Moselle (2002), who also show that legislators may prefer to make proposals for the two dimensions together despite separable preferences, but their model does not have asymmetric information or communication. We would also like to point out that the result is sensitive to the nature of the uncertainty. In a related paper (Chen and Eraslan 2013), we show that bundled bargaining may result in informational loss when ideological positions, rather than intensities, are private information, and in that case, it may be better for the chair to bargain the two dimensions separately.

5.5 *Extension to more than three players*

We have considered a multilateral bargaining game with three players. In the Supplementary Appendix, we take a first step in generalizing our analysis to more than three players. Specifically, we analyze an extension in which there are $n \geq 3$ legislators other than the chair, but only two legislators have private information.[28] The main results derived in the three-player case are robust. In particular, in the extension with n legislators, a legislator can still convey limited information of whether he will fight or compromise. Under majority rule, any legislator who holds a position sufficiently distant from the chair's (specifically, a position further away from the chair's than the median position is) cannot be informative. There are equilibria in which more than one legislator is informative, but this happens only when their positions are weakly closer to the chair's than the median position is to the chair's.

6. Concluding remarks

In this paper, we develop a new model of legislative bargaining that incorporates private information about preferences and allows speech making before a bill is proposed. Although the model is simple, our analysis generates interesting predictions about what speeches can be credible even without commitment, and how they influence proposals and legislative outcomes.

We believe that both private information and communication are essential elements of the legislative decision making process. Our paper has taken a first step in understanding their roles in legislative bargaining. There are many more issues to explore and many ways to extend our model, and what follows is a brief discussion of some of them.

[28]Suppose that at least κ votes other than the chair's are needed for a proposal to pass and that only two legislators have private information. We show that if the prior satisfies IHRP, then any proposal elicited in a connected equilibrium gives transfers to at most κ legislators, and if a privately informed legislator is included in a proposal, then he has veto power with respect to that proposal. We then show that Propositions 3 and 4(ii) generalize to this extension. Although a full analysis of the more general case in which more than two legislators have private information is beyond the scope of this paper, we still find the extension in the Supplementary Appendix useful because it applies to a legislature with a low turnover so that the preferences of most legislators are known through past experience and only a few new members may have private information on their preferences.

Our motivation for incorporating private information into legislative bargaining is that individual legislators know their preferences better than others. Another possible source of private information is expertise (perhaps acquired through specialized committee work or from staff advisors) regarding the consequences of policies. Although the role of this kind of "common value" private information in legislative decision making has been studied (e.g., Austen-Smith 1990), it is only in the context of one-dimensional spatial policy making. It would be interesting to explore it further when there is trade-off between ideology and distribution.

In our model, the chair does not have private information about her preference, consistent with the observation that bill proposers are typically established members with known positions. Sometimes, however, legislators can be uncertain about the leaders' goals and, in particular, how much compromise the leaders are willing to make in exchange for their votes. In this case, the proposal put on the table may also reveal some of the proposer's private information. This kind of signaling effect becomes especially relevant when the legislators have interdependent preferences or when the proposal is not an ultimatum, but can be modified if agreement fails.

We have considered a specific extensive form in which the legislators send messages simultaneously. It would be interesting to explore whether and how some of our results change if the legislators send public messages sequentially. In that case, the design of the optimal order of speeches (from the perspective of the proposer as well as the legislature) itself is an interesting question. Another design question with respect to communication protocol is whether the messages should be public or private. Although this distinction does not matter for the model in this paper because we assume simultaneous speeches and one round of bargaining, it would matter if either there were multiple rounds of bargaining or the preferences were interdependent.

Appendix

Proof of Lemma 1. (i) Since type θ_1 weakly prefers z' to z, we have $x_1' + \theta_1 v(y', \hat{y}_1) \geq x_1 + \theta_1 v(y, \hat{y}_1)$, which implies that $x_1' - x_1 \geq \theta_1(v(y, \hat{y}_1) - v(y', \hat{y}_1))$.

Suppose $v(y, \hat{y}_1) - v(y', \hat{y}_1) \leq 0$. Since $x_1' - x_1 > 0$ and $\theta_1' > 0$, it follows that $x_1' - x_1 > 0 \geq \theta_1'(v(y, \hat{y}_1) - v(y', \hat{y}_1))$, i.e., $x_1' + \theta_1' v(y', \hat{y}_1) > x_1 + \theta_1' v(y, \hat{y}_1)$.

Suppose $v(y, \hat{y}_1) - v(y', \hat{y}_1) > 0$. Then $\theta_1(v(y, \hat{y}_1) - v(y', \hat{y}_1)) > \theta_1'(v(y, \hat{y}_1) - v(y', \hat{y}_1))$ since $\theta_1' < \theta_1$. Hence $x_1' - x_1 > \theta_1'(v(y, \hat{y}_1) - v(y', \hat{y}_1))$, i.e., $x_1' + \theta_1' v(y', \hat{y}_1) > x_1 + \theta_1' v(y, \hat{y}_1)$.

(ii) Since type θ_1 weakly prefers z'' to z, we have $x_1'' + \theta_1 v(y'', \hat{y}_1) \geq x_1 + \theta_1 v(y, \hat{y}_1)$, which implies that $\theta_1(v(y'', \hat{y}_1) - v(y, \hat{y}_1)) \geq x_1 - x_1''$. Since $x_1 - x_1'' > 0$, we have $v(y'', \hat{y}_1) - v(y, \hat{y}_1) > 0$. So, for $\theta_1'' > \theta_1$, we have $\theta_1''(v(y'', \hat{y}_1) - v(y, \hat{y}_1)) > \theta_1(v(y'', \hat{y}_1) - v(y, \hat{y}_1)) \geq x_1 - x_1''$, i.e., $x_1'' + \theta_1'' v(y'', \hat{y}_1) > x_1 + \theta_1'' v(y, \hat{y}_1)$. □

Proof of Lemma 2. Suppose z' and z'' are elicited in equilibrium (μ, γ, π) and $x_1' > 0$, $x_1'' > 0$. For any z, let $\alpha(z)$ be the set of types who elicit z and accept it. Formally, $\alpha(z) = \{\theta_1 : \pi(\mu_1(\theta_1)) = z \text{ and } \gamma_1(z, \theta_1) = 1\}$. Since any proposal elicited in an equilibrium is accepted by some type who elicits it, $\alpha(z') \neq \varnothing$ and $\alpha(z'') \neq \varnothing$. Let $\theta_1' = \sup \alpha(z')$ and $\theta_1'' = \sup \alpha(z'')$. Also, let $o(\theta_1) = \pi(\mu_1(\theta_1))$ if $\gamma_1(\mu_1(\theta_1), \theta_1) = 1$ and let $o(\theta_1) = s$ otherwise.

Let $u_1^e(\theta_1)$ be type θ_1's payoff in (μ, γ, π). Formally, $u_1^e(\theta_1) = u_1(o(\theta_1), \theta_1)$. Note that $u_1^e(\theta_1) = u_1(z, \theta_1)$ for any $\theta_1 \in \alpha(z)$.

Claim 1. *We have* $u_1^e(\theta_1') = u_1(z', \theta_1') = u_1(s, \theta_1')$ *and* $u_1^e(\theta_1'') = u_1(z'', \theta_1'') = u_1(s, \theta_1'')$.

Proof. We show that $u_1^e(\theta_1') = u_1(z', \theta_1') = u_1(s, \theta_1')$. Similar arguments apply to θ_1''.

To show that $u_1^e(\theta_1') = u_1(z', \theta_1')$, first note that $u_1^e(\theta_1') \geq u_1(z', \theta_1')$ since type θ_1' can elicit z' and accept it. Suppose $u_1^e(\theta_1') > u_1(z', \theta_1')$. Since $u_1(o(\theta_1'), \theta_1) - u_1(z', \theta_1)$ is continuous in θ_1, there exists $\theta_1 \in \alpha(z')$ sufficiently close to θ_1' such that $u_1(o(\theta_1'), \theta_1) > u_1(z', \theta_1)$. This contradicts that $u_1^e(\theta_1) = u_1(z', \theta_1)$ for any $\theta_1 \in \alpha(z')$.

To show that $u_1^e(\theta_1') = u_1(s, \theta_1')$, first note that for any θ_1, $u_1^e(\theta_1) \geq u_1(s, \theta_1)$ since type θ_1 can reject the proposal it elicits. Suppose $u_1^e(\theta_1') > u_1(s, \theta_1')$. Since $u_1^e(\theta_1') = u_1(z', \theta_1')$, we have $u_1(z', \theta_1') > u_1(s, \theta_1')$. Since $x_1' > 0$, there exists $\hat{z} = (\hat{y}; \hat{x})$ with $\hat{y} = y'$ and $\hat{x}_1 \in (0, x_1')$ such that $u_1(\hat{z}, \theta_1') > u_1(s, \theta_1')$. Since $\theta_1 < \theta_1'$ for any $\theta_1 \in \alpha(z')$, it follows from Lemma 1 that $u_1(\hat{z}, \theta_1) > u_1(s, \theta_1)$ and $\gamma_1(\hat{z}, \theta_1) = 1$ for any $\theta_1 \in \alpha(z')$. Since $u_0(\hat{z}) > u_0(z')$, this contradicts the optimality of z'. ◁

We next show that $z' = z''$. Suppose not. Consider the following two possibilities. (a) Suppose $x_1' = x_1''$ and without loss of generality, $y' < y''$. Recall from footnote 14 that $y'' \leq e(\hat{y}_1)$ and, therefore, $y' < y'' \leq e(\hat{y}_1) \leq \hat{y}_1$. We have $u_1(z', \theta_1) < u_1(z'', \theta_1)$ for all $\theta_1 \in \Theta_1$ since $x_1' = x_1''$ and $u_1(z, \theta_1)$ is increasing in y for $y < \hat{y}_1$, contradicting that $\alpha(z') \neq \varnothing$. (b) Suppose $x_1' \neq x_1''$ and, without loss of generality, $x_1' > x_1''$. By definition of $\alpha(z)$, for all $\theta_1 \in \alpha(z')$, $u_1^e(\theta_1) = u_1(z', \theta_1) \geq u_1(z'', \theta_1)$, and for all $\theta_1 \in \alpha(z'')$, $u_1^e(\theta_1) = u_1(z'', \theta_1) \geq u_1(z', \theta_1)$. By Lemma 1, any type in $\alpha(z')$ is strictly lower than any type in $\alpha(z'')$ and, therefore, $\theta_1' < \theta_1''$. Since $u_1(z'', \theta_1'') = u_1(s, \theta_1'')$ by Claim 1 and $x_1'' > 0$, Lemma 1 implies that $u_1(z'', \theta_1') > u_1(s, \theta_1')$. This contradicts $u_1^e(\theta_1') = u_1(s, \theta_1')$ since type θ_1' could elicit z'' and obtain a strictly higher payoff than his equilibrium payoff. □

Proof of Proposition 2. For any $z \in Y \times X$ and $\theta_1', \theta_1'' \in \Theta_1$ with $\theta_1' < \theta_1''$, let $\tau(z, \theta_1', \theta_1'')$ denote the highest type in $[\theta_1', \theta_1'']$ who weakly prefers z to the status quo if such a type exists, and let $\tau(z, \theta_1', \theta_1'') = \theta_1'$ otherwise. Formally,

$$\tau(z, \theta_1', \theta_1'') = \begin{cases} \max\{\theta_1 \in [\theta_1', \theta_1''] : u_1(z, \theta_1) \geq u_1(s, \theta_1)\} & \text{if } u_1(z, \theta_1') \geq u_1(s, \theta_1') \\ \theta_1' & \text{otherwise.} \end{cases}$$

Let $k(\theta_1', \theta_1'')$ be the set of optimal proposals for the chair if she knows that $\theta_1 \in [\theta_1', \theta_1'']$. That is,

$$k(\theta_1', \theta_1'') = \operatorname*{arg\,max}_{z \in Y \times X} u_0(z)\big[F_1(\tau(z, \theta_1', \theta_1'')) - F_1(\theta_1')\big] + u_0(s)\big[F_1(\theta_1'') - F_1(\tau(z, \theta_1', \theta_1''))\big].$$

Let $k(\theta_1, \theta_1) = \{z^1(\theta_1)\}$. We first establish that if the no-transfer proposal z^{NT} is optimal when the chair's belief about legislator 1's types is on some interval, then it is also optimal when her belief is on a "higher interval," and if z^{NT} is not uniquely optimal when the chair's belief about legislator 1's types is on some interval, then it is also not uniquely optimal when her belief is on a "lower interval."

Claim 2. *Let* $\theta'_l, \theta''_l, \theta'_h, \theta''_h \in \Theta_1$ *be such that* $\theta'_l < \theta''_l$, $\theta'_h < \theta''_h$, $\theta'_l \leq \theta'_h$, *and* $\theta''_l \leq \theta''_h$. *(i) If* $z^{\mathrm{NT}} \in k(\theta'_l, \theta''_l)$, *then* $z^{\mathrm{NT}} \in k(\theta'_h, \theta''_h)$. *(ii) If* $k(\theta'_h, \theta''_h) \neq \{z^{\mathrm{NT}}\}$, *then* $k(\theta'_l, \theta''_l) \neq \{z^{\mathrm{NT}}\}$.

Proof. For any $z \in Y \times X$, let $p(z)$ be the probability that z is accepted conditional on legislator 1's type being in $[\theta'_h, \theta''_h]$ and let $r(z)$ be the probability that z is accepted conditional on legislator 1's type being in $[\theta'_l, \theta''_l]$. Formally,

$$p(z) = \frac{F_1(\tau(z, \theta'_h, \theta''_h)) - F_1(\theta'_h)}{F_1(\theta''_h) - F_1(\theta'_h)}$$

and

$$r(z) = \frac{F_1(\tau(z, \theta'_l, \theta''_l)) - F_1(\theta'_l)}{F_1(\theta''_l) - F_1(\theta'_l)}.$$

Note that $r(z^{\mathrm{NT}}) = p(z^{\mathrm{NT}}) = 1$. We first show that $r(z) \geq p(z)$ for any z. If $\tau(z, \theta'_l, \theta''_l) = \theta''_l$, then $r(z) = 1 \geq p(z)$. If $\tau(z, \theta'_h, \theta''_h) = \theta'_h$, then $p(z) = 0 \leq r(z)$. If $\tau(z, \theta'_l, \theta''_l) < \theta''_l$ and $\tau(z, \theta'_h, \theta''_h) > \theta'_h$, consider the following two possibilities. Suppose $\tau(z, \theta'_h, \theta''_h) < \theta''_l$. Then $\tau(z, \theta'_h, \theta''_h) = \tau(z, \theta'_l, \theta''_l)$. Suppose $\tau(z, \theta'_h, \theta''_h) \geq \theta''_l$. Note that since $\tau(z, \theta'_h, \theta''_h) > \theta'_h$, there exists some type higher than θ'_h who weakly prefers $z = (y; x)$ to s. If $x_1 = 0$, then every type of legislator 1 weakly prefers z to s and, therefore, $\tau(z, \theta'_l, \theta''_l) = \theta''_l$, a contradiction. If $x_1 > 0$, then by Lemma 1, type θ''_l strictly prefers z to s and, therefore, $\tau(z, \theta'_l, \theta''_l) = \theta''_l$, a contradiction. Hence $\tau(z, \theta'_l, \theta''_l) = \tau(z, \theta'_h, \theta''_h)$ and $r(z) \geq (F_1(\tau(z, \theta'_l, \theta''_l)) - F_1(\theta'_l))/(F_1(\theta''_h) - F_1(\theta'_l)) \geq p(z)$.

Part (i): If $z^{\mathrm{NT}} \in k(\theta'_l, \theta''_l)$, then $u_0(z^{\mathrm{NT}}) \geq u_0(z)r(z) + u_0(s)(1 - r(z))$ for any $z \in Y \times X$. Since $r(z) \geq p(z)$, for any z such that $u_0(z) \geq u_0(s)$, we have $u_0(z)r(z) + u_0(s)(1 - r(z)) \geq u_0(z)p(z) + u_0(s)(1 - p(z))$. Hence $u_0(z^{\mathrm{NT}}) \geq u_0(z)p(z) + u_0(s)(1 - p(z))$, which implies that $z^{\mathrm{NT}} \in k(\theta'_h, \theta''_h)$.

Part (ii): If $k(\theta'_h, \theta''_h) \neq \{z^{\mathrm{NT}}\}$, then there exists $z \neq z^{\mathrm{NT}}$ such that $u_0(z) \geq u_0(s)$ and $u_0(z)p(z) + u_0(s)(1 - p(z)) \geq u_0(z^{\mathrm{NT}})$. Since $r(z) \geq p(z)$, we have $u_0(z)r(z) + u_0(s)(1 - r(z)) \geq u_0(z^{\mathrm{NT}})$ and, therefore, $k(\theta'_l, \theta''_l) \neq \{z^{\mathrm{NT}}\}$. ◁

The "if" part: Let $t'_1 = \sup\{\theta_1 \in [\underline{\theta}_1, \overline{\theta}_1]$ such that $k(\underline{\theta}_1, \theta_1) \neq \{z^{\mathrm{NT}}\}\}$ and let $t''_1 = \inf\{\theta_1 \in [\underline{\theta}_1, \overline{\theta}_1]$ such that $z^{\mathrm{NT}} \in k(\theta_1, \overline{\theta}_1)\}$. Under conditions (i) and (ii) in Proposition 2, t'_1 and t''_1 are well defined. Note that $k(\theta_1, \overline{\theta}_1)$ is an upper hemicontinuous correspondence by the Theorem of the Maximum, which implies that $z^{\mathrm{NT}} \in k(t''_1, \overline{\theta}_1)$. We next show that $t'_1 \geq t''_1$. Suppose to the contrary $t'_1 < t''_1$. Then there exists $\theta_1 \in (t'_1, t''_1)$ such that $k(\underline{\theta}_1, \theta_1) = \{z^{\mathrm{NT}}\}$ and $z^{\mathrm{NT}} \notin k(\theta_1, \overline{\theta}_1)$, contradicting Claim 2. Hence $t'_1 \geq t''_1$. We next construct a size-2 equilibrium. Fix $\tilde{\theta}_1 \in [t''_1, t'_1]$. By Claim 2, $k(\underline{\theta}_1, \tilde{\theta}_1) \neq \{z^{\mathrm{NT}}\}$ and $z^{\mathrm{NT}} \in k(\tilde{\theta}_1, \overline{\theta}_1)$. Fix $m_1^c, m_1^f \in M_1$, and let $\mu_1(\theta_1) = m_1^c$ if $\theta_1 < \tilde{\theta}_1$ and $\mu_1(\theta_1) = m_1^f$ if $\theta_1 \geq \tilde{\theta}_1$, $\pi(m_1^c) \in k(\underline{\theta}_1, \tilde{\theta}_1) \setminus \{z^{\mathrm{NT}}\}$, $\pi(m_1^f) = z^{\mathrm{NT}}$, and $\pi(m_1) \in \{\pi(m_1^c), \pi(m_1^f)\}$ for any other $m_1 \in M_1$. Also, let γ_1 satisfy (E1). Since the constructed (μ, γ, π) is an equilibrium profile, a size-2 equilibrium exists.

The "only if" part: Suppose a size-2 equilibrium (μ, γ, π) exists. By Proposition 1, two proposals z^{NT} and $z = (y; x)$ are elicited in this equilibrium, where $y < e(y_1)$ and $x_1 > 0$.

Since z is elicited in this equilibrium, there exists a type $\theta_1 \in \Theta_1$ such that $u_1(z, \theta_1) \geq u_1(s, \theta_1)$. Since $x_1 > 0$, by Lemma 1, we have $u_1(z, \underline{\theta}_1) \geq u_1(s, \underline{\theta}_1)$. Note also that since z is optimal for the chair under some belief and z^{NT} is accepted by all types θ_1, we have $u_0(z) \geq u_0(z^{\mathrm{NT}})$. It follows that if the chair is sure that legislator 1's type is $\underline{\theta}_1$, then it is better to propose z than z^{NT} and, therefore, $z^1(\underline{\theta}_1) \neq z^{\mathrm{NT}}$.

Since z^{NT} is elicited in this equilibrium, there exists a type $\theta_1 \in \Theta_1$ such that $z^1(\theta_1) = z^{\mathrm{NT}}$. As shown in Section 3, if $z^1(\theta_1) = z^{\mathrm{NT}}$, then $z^1(\theta_1') = z^{\mathrm{NT}}$ for all $\theta_1' \geq \theta_1$. Hence $z^1(\overline{\theta}_1) = z^{\mathrm{NT}}$. □

Proof of Lemma 3. Fix a connected equilibrium (μ, γ, π). Consider any message profile m sent in this equilibrium. We show below that $\pi(m_1, m_2) = z^* = (y^*; x^*)$ is not a two-transfer proposal.

Case (i). Suppose $\mu_i^{-1}(m_i)$ is a singleton for some $i \in \{1, 2\}$. Without loss of generality, suppose $\mu_1^{-1}(m_1)$ is a singleton and let $\theta_1 = \mu_1^{-1}(m_1)$. If $u_1(z^*, \theta_1) \geq u_1(s, \theta_1)$, then type θ_1 accepts z^* and we must have $x_2^* = 0$. If $u_1(z^*, \theta_1) < u_1(s, \theta_1)$, then type θ_1 rejects z^* and we must have $x_1^* = 0$.

Case (ii). Suppose $\mu_i^{-1}(m_i)$ is a nondegenerate interval for $i = 1, 2$. Let G_i be the posterior distribution function of the chair when receiving m_i and let g_i be the associated density. Recall that for any z, $t_1(z)$ denotes the highest type of legislator 1 willing to accept z. Define $t_2(z)$ analogously. Let $\beta(z) = 1 - (1 - G_1(t_1(z)))(1 - G_2(t_2(z)))$ and $d = x_1^* + x_2^*$. Consider the problem

$$\max_{x \in X} (c - d + \theta_0 v(y^*, \hat{y}_0))\beta(y^*; x) + \theta_0 v(\tilde{y}, \hat{y}_0)(1 - \beta(y^*; x)) \tag{2}$$

subject to $x_1 + x_2 = d$. Since z^* is an optimal proposal when the chair receives m, x^* is a solution to (2).

We can rewrite the optimization problem (2) as

$$\max_{x \in X} a^* \beta(y^*; x) + b^*$$

subject to $x_1 + x_2 = d$, where $a^* = c - d + \theta_0 v(y^*, \hat{y}_0) - \theta_0 v(\tilde{y}, \hat{y}_0)$ and $b^* = \theta_0 v(\tilde{y}, \hat{y}_0)$. Since z^* is an optimal proposal and the chair can guarantee a payoff of $c + \theta_0 v(\tilde{y}, \hat{y}_0)$ by proposing $(\tilde{y}; c, 0, 0)$, it follows that $a^* > 0$. Since a^* and b^* are constant in x, x^* maximizes the probability of acceptance $\beta(y^*, x)$. Equivalently, x^* minimizes the probability of rejection $1 - \beta(y^*, x) = (1 - G_1(t_1(y^*, x)))(1 - G_2(t_2(y^*, x)))$.

Suppose, toward a contradiction, that $x_1^* > 0$ and $x_2^* > 0$. Let $v_i^* = v(\tilde{y}, \hat{y}_i) - v(y^*, \hat{y}_i)$. Since $x_i^* > 0$, it follows that $v_i^* > 0$ and we can write $t_i(y^*, x) = x_i / v_i^*$. Since x^* minimizes $1 - \beta(y^*, x)$, x_1^* solves the problem

$$\min_{0 \leq x_1 \leq d} \ln\bigl(1 - G_1(x_1/v_1^*)\bigr) + \ln(1 - G_2((d - x_1)/v_2^*)). \tag{3}$$

By Corollary 5 in Bagnoli and Bergstrom (2005), a truncation of a distribution preserves IHRP. Since F_i satisfies IHRP, G_i satisfies IHRP, implying that $\ln(1 - G_i)$ is strictly concave for $i = 1, 2$. It follows, at each $v_i^* > 0$, that the objective function of (3) is strictly concave in x_1. Hence, the solution to (3) cannot be interior, contradicting that $x_i^* \in (0, d)$. □

Proof of Lemma 5. Suppose, to the contrary, that there exists a type θ_j' such that $u_j(z, \theta_j') \geq u_j(s, \theta_j')$. Since $x_j = 0$, this implies that $v(y, \hat{y}_j) \geq v(\tilde{y}, \hat{y}_j)$ and, therefore, $u_j(z, \theta_j) \geq u_j(s, \theta_j)$ for all $\theta_j \in \Theta_j$. Consider $z' = (y'; x')$ with $y' = y$ and $x_i' = x_j' = 0$. We have $u_j(z', \theta_j) \geq u_j(s, \theta_j)$ and $\gamma_j(z', \theta_j) = 1$ for all $\theta_j \in \Theta_j$. Since $x_i' < x_i$, we have $u_0(z') > u_0(z)$, contradicting the optimality of z. Hence, every type of legislator j rejects z and legislator i is pivotal. Since the chair does not make any proposal that is rejected with probability 1, it follows that $u_i(z, \theta_i) \geq u_i(s, \theta_i)$ for some $\theta_i \in \Theta_i$. □

Proof of Proposition 3. To give an outline of the proof, we first show in Lemma 7 that in (μ, γ, π), there exists at most one m_i sent by a positive measure of θ_i such that $q_i(m_i) > 0$; when such a message exists, the types who send this message form an interval at the lower end of Θ_i. We also show that there exists at most one message m_i sent by a single type such that $q_i(m_i) > 0$. So at most two m_i's have the property that $q_i(m_i) > 0$ and one of them is sent by only a single type. We then show in Claim 5 that if legislator i is informative, then a positive measure of types send a message that results in a positive probability of inclusion and a positive measure of types send messages that result in zero probability of inclusion. We then show that legislator i's message rule is equivalent to a size-2 message rule that satisfies the properties described in Proposition 3.

Recall that $V_i(m_i, \theta_i)$ is type θ_i's expected payoff from sending m_i in (μ, γ, π). For any $\theta_i \in \Theta_i$, $m_i \in M_i$, and interval $T \subseteq \Theta_j$, let $V_i(m_i, \theta_i | T)$ be type θ_i's expected payoff if he sends m_i, conditional on $\theta_j \in T$. The proof of Lemma 7 makes use of the following lemma.

Lemma 6. *Suppose F_1 and F_2 satisfy IHRP. Fix a simple connected equilibrium (μ, γ, π) in $\Gamma^{\{1,2\}}$. Suppose that $m = (\mu_1(\theta_1), \mu_2(\theta_2))$ for some (θ_1, θ_2), and that $\pi(m) = z = (y; x)$ with $x_i > 0$. Let $\theta_i^s = \sup\{\theta_i : \mu_i(\theta_i) = m_i\}$. Then (i) $u_i(s, \theta_i^s) \geq u_i(z, \theta_i^s)$ and (ii) $V_i(m_i, \theta_i^s | \mu_j^{-1}(m_j)) = u_i(s, \theta_i^s)$, i.e., if type θ_i^s elicits z followed by his optimal acceptance rule, he receives a payoff equal to $u_i(s, \theta_i^s)$.*

Proof. Suppose, to the contrary, that $u_i(z, \theta_i^s) > u_i(s, \theta_i^s)$. Since $x_i > 0$, there exists $\varepsilon > 0$ and $\hat{z} = (y; x_0 + \varepsilon, x_i - \varepsilon, x_j)$ such that $u_i(\hat{z}, \theta_i^s) > u_i(s, \theta_i^s)$. Since $\theta_i \leq \theta_i^s$ for any $\theta_i \in \mu_i^{-1}(m_i)$, by Lemma 1, $\hat{z}$ is accepted by any $\theta_i \in \mu_i^{-1}(m_i)$. Since $u_0(\hat{z}) > u_0(z)$, this contradicts the optimality of z. So $u_i(s, \theta_i^s) \geq u_i(z, \theta_i^s)$. If $u_i(z, \theta_i^s) = u_i(s, \theta_i^s)$, then $\gamma_i(z, \theta_i^s) = 1$. If $u_i(z, \theta_i^s) < u_i(s, \theta_i^s)$, then $\gamma_i(z, \theta_i^s) = 0$. Since $x_i > 0$, by Lemmas 3 and 5, legislator i is pivotal with respect to z. Therefore, $V_i(m_i, \theta_i^s | \mu_j^{-1}(m_j)) = \max\{u_i(z, \theta_i^s), u_i(s, \theta_i^s)\} = u_i(s, \theta_i^s)$. ◁

Lemma 7. *Suppose F_1 and F_2 satisfy IHRP. Fix a simple connected equilibrium (μ, γ, π) in $\Gamma^{\{1,2\}}$. Let $\theta_i' < \theta_i''$, $m_i' = \mu_i(\theta_i')$, and $m_i'' = \mu_i(\theta_i'')$. Suppose $q_i(m_i'') > 0$. (i) If $\mu_i^{-1}(m_i'')$ is not a singleton, then $m_i' = m_i''$. (ii) If $\mu_i^{-1}(m_i'')$ is a singleton, then $\mu_i^{-1}(m_i')$ is not a singleton.*

Proof. Part (i). Suppose, to the contrary, that $m_i' \neq m_i''$. Let $\theta_i^l = \sup \mu_i^{-1}(m_i')$ and $\theta_i^h = \sup \mu_i^{-1}(m_i'')$. We first establish Claim 3, which allows us to partition Θ_j into three sets. Next, we compare the expected payoffs of types θ_i^l and θ_i^h from sending messages m_i' and m_i'', conditional on θ_j being in each of these sets. Finally, we show in Claim 4 that either type θ_i^l or type θ_i^h has a profitable deviation, leading to a contradiction.

Claim 3. *For any m_j sent by some $\theta_j \in \Theta_j$, if $\pi(m_i'', m_j)$ includes i, then $\pi(m_i', m_j)$ also includes i. Hence $q_i(m_i') \geq q_i(m_i'')$.*

Proof. Suppose $\pi(m_i'', m_j) = (y''; x'')$ includes i. By Lemmas 3 and 5, legislator i is pivotal with respect to $\pi(m_i'', m_j)$. Since $\pi(m_i'', m_j)$ is accepted with positive probability and $\mu_i^{-1}(m_i'')$ is a nondegenerate interval, we have $P_i(\theta_i \in \mu_i^{-1}(m_i'') | u_i(\pi(m_i'', m_j), \theta_i) \geq u_i(s, \theta_i)) > 0$. By Lemma 1, if $u_i(\pi(m_i'', m_j), \theta_i) \geq u_i(s, \theta_i)$, then $u_i(\pi(m_i'', m_j), \tilde{\theta}_i) > u_i(s, \tilde{\theta}_i)$ for all $\tilde{\theta}_i < \theta_i$. Hence $P_i(\theta_i \in \mu_i^{-1}(m_i'') | u_i(\pi(m_i'', m_j), \theta_i) > u_i(s, \theta_i)) > 0$.

Given any $\varepsilon \in (0, x_i'')$, let $z_\varepsilon = (y''; x_0'' + \varepsilon, x_i'' - \varepsilon, x_j'')$. Since $x_i'' > 0$ and u_i is continuous in x_i, it follows that for ε sufficiently small, $P_i(\theta_i \in \mu_i^{-1}(m_i'') | u_i(z_\varepsilon, \theta_i) > u_i(s, \theta_i)) > 0$ and, therefore, $u_i(z_\varepsilon, \theta_i) > u_i(s, \theta_i)$ if $\theta_i = \inf\{\mu_i^{-1}(m_i'')\}$.

Since $\theta_i' < \theta_i''$, $m_i' \neq m_i''$ and (μ, γ, π) is a connected equilibrium, $\sup\{\mu_i^{-1}(m_i')\} \leq \inf\{\mu_i^{-1}(m_i'')\}$. Let $\varepsilon \in (0, x_i'')$ be such that $u_i(z_\varepsilon, \theta_i) > u_i(s, \theta_i)$ for $\theta_i = \inf\{\mu_i^{-1}(m_i'')\}$. By Lemma 1, $u_i(z_\varepsilon, \theta_i) > u_i(s, \theta_i)$ for all $\theta_i \in \mu_i^{-1}(m_i')$. Since $u_0(z_\varepsilon) > u_0(\pi(m_i'', m_j))$ and z_ε is accepted by all $\theta_i \in \mu_i^{-1}(m_i')$, we have $U_0(H(m_i', m_j)) > U_0(H(m_i'', m_j))$. Since $\pi(m_i'', m_j)$ includes i, it follows that $U_0(H(m_i'', m_j)) \geq U_0^{-i}(H_j(m_j))$. So $U_0(H(m_i', m_j)) > U_0(H(m_i'', m_j)) \geq U_0^{-i}(H_j(m_j))$, implying that $\pi(m_i', m_j)$ includes i. It immediately follows that $q_i(m_i') \geq q_i(m_i'')$. ◁

By Claim 3, we can partition Θ_j into A, B, and C where $A = \{\theta_j \in \Theta_j \mid$ both $\pi(m_i', \mu_j(\theta_j))$ and $\pi(m_i'', \mu_j(\theta_j))$ exclude $i\}$, $B = \{\theta_j \in \Theta_j \mid \pi(m_i', \mu_j(\theta_j))$ includes i and $\pi(m_i'', \mu_j(\theta_j))$ excludes $i\}$, and $C = \{\theta_j \in \Theta_j \mid$ both $\pi(m_i', \mu_j(\theta_j))$ and $\pi(m_i'', \mu_j(\theta_j))$ include $i\}$. Claim 3 also implies that $C = \{\theta_j \in \Theta_j \mid \pi(m_i'', \mu_j(\theta_j))$ include $i\}$, and $P_j(C) = q_i(m_i'')$. Since $q_i(m_i'') > 0$, we have $P_j(C) > 0$.

Recall that $\theta_i^l = \sup \mu_i^{-1}(m_i')$ and $\theta_i^h = \sup \mu_i^{-1}(m_i'')$. Since $\mu_i^{-1}(m_i'')$ is not a singleton and $m_i' \neq m_i''$, we have $\theta_i^h > \theta_i^l$.

If $\theta_j \in A$, then both $\pi(m_i', \mu_j(\theta_j))$ and $\pi(m_i'', \mu_j(\theta_j))$ exclude i. In an SCE, it follows that $\pi(m_i', \mu_j(\theta_j)) = \pi(m_i'', \mu_j(\theta_j))$ and so $V_i(m_i', \theta_i | A) = V_i(m_i'', \theta_i | A)$ for all θ_i.

If $\theta_j \in B$, then $\pi(m_i', \mu_j(\theta_j))$ includes i. By Lemmas 3 and 5, legislator i is pivotal with respect to $\pi(m_i', \mu_j(\theta_j))$. By Lemma 6, $u_i(s, \theta_i^l) \geq u_i(\pi(m_i', \mu_j(\theta_j)), \theta_i^l)$. Since legislator i can achieve his status quo payoff by rejecting $\pi(m_i', \mu_j(\theta_j))$, it follows that $V_i(m_i', \theta_i^l | B) = u_i(s, \theta_i^l)$. Since $\theta_i^h > \theta_i^l$, by Lemma 1, $u_i(s, \theta_i^h) > u_i(\pi(m_i', \mu_j(\theta_j)), \theta_i^h)$. Hence, we also have $V_i(m_i', \theta_i^h | B) = u_i(s, \theta_i^h)$.

If $\theta_j \in C$, then both $\pi(m_i', \mu_j(\theta_j))$ and $\pi(m_i'', \mu_j(\theta_j))$ include i. By the same argument as for the case when $\theta_j \in B$, $V_i(m_i', \theta_i^l | C) = u_i(s, \theta_i^l)$, $V_i(m_i', \theta_i^h | C) = u_i(s, \theta_i^h)$, and $V_i(m_i'', \theta_i^h) = u_i(s, \theta_i^h)$. Also, since $\pi(m_i'', \mu_j(\theta_j))$ is accepted with positive probability, there exists a type $\theta_i > \theta_i^l$ such that $u_i(\pi(m_i'', \mu_j(\theta_j)), \theta_i) \geq u_i(s, \theta_i)$. By Lemma 1, $u_i(\pi(m_i'', \mu_j(\theta_j)), \theta_i^l) > u_i(s, \theta_i^l)$. Hence, $V_i(m_i'', \theta_i^l | C) > u_i(s, \theta_i^l)$.

In what follows, we show that if $V_i(m'_i, \theta^l_i) \geq V_i(m''_i, \theta^l_i)$, then $V_i(m'_i, \theta^h_i) > V_i(m''_i, \theta^h_i)$, which implies that either type θ^l_i or type θ^h_i has a strictly profitable deviation.

Claim 4. *Suppose* $V_i(m'_i, \theta^l_i) \geq V_i(m''_i, \theta^l_i)$. *Then* $V_i(m'_i, \theta^h_i) > V_i(m''_i, \theta^h_i)$.

Proof. Since $V_i(m'_i, \theta^l_i | A \cup C) < V_i(m''_i, \theta^l_i | A \cup C)$ and $P_j(C) > 0$, it follows from $V_i(m'_i, \theta^l_i) \geq V_i(m''_i, \theta^l_i)$ that $P_j(B) > 0$ and $V_i(m'_i, \theta^l_i | B) > V_i(m''_i, \theta^l_i | B)$.

Note that for any z and z' that both exclude i, if $u_i(z, \theta_i) > u_i(z', \theta_i)$ for some $\theta_i \in \Theta_i$, then $u_i(z, \theta_i) > u_i(z', \theta_i)$ for all $\theta_i \in \Theta_i$. As shown in the previous paragraph, $V_i(m'_i, \theta^l_i | B) > V_i(m''_i, \theta^l_i | B)$. Since $V_i(m'_i, \theta^l_i | B) = u_i(s, \theta^l_i)$, $V_i(m''_i, \theta^l_i | B) = u_i(\pi(m''_i, \mu_j(\theta_j)), \theta^l_i)$ for $\theta_j \in B$, and both s and $\pi(m''_i, \mu_j(\theta_j))$ exclude i when $\theta_j \in B$, it follows that $u_i(s, \theta^h_i) > V_i(m''_i, \theta^h_i | B)$. Since $V_i(m'_i, \theta^h_i | B) = u_i(s, \theta^h_i)$, we have $V_i(m'_i, \theta^h_i | B) > V_i(m''_i, \theta^h_i | B)$. Since $V_i(m'_i, \theta^h_i | A \cup C) = V_i(m''_i, \theta^h_i | A \cup C)$ and $P_j(B) > 0$, it follows that $V_i(m'_i, \theta^h_i) > V_i(m''_i, \theta^h_i)$. ◁

Since Claim 4 implies that either type θ^l_i or type θ^h_i has a strictly profitable deviation, we have a contradiction. Hence $m''_i = m'_i$.

Part (ii). The proof is similar to that of part (i). Suppose, to the contrary, that $\mu_i^{-1}(m'_i)$ is a singleton. As in part (i), we first prove the claim that for any m_j sent by some $\theta_j \in \Theta_j$, if $\pi(m''_i, m_j)$ includes i, then $\pi(m'_i, m_j)$ also includes i. To show this, suppose $\pi(m''_i, m_j)$ includes i. Since $\mu_i^{-1}(m''_i)$ is a singleton, $\pi(m''_i, m_j)$ is accepted by θ''_i, i.e., $u_i(\pi(m''_i, m_j), \theta''_i) \geq u_i(s, \theta''_i)$. Since $\theta'_i < \theta''_i$, by Lemma 1, $u_i(\pi(m''_i, m_j), \theta'_i) > u_i(s, \theta'_i)$. Given any $\varepsilon \in (0, x''_i)$, let $z_\varepsilon = (y''; x''_0 + \varepsilon, x''_i - \varepsilon, x''_j)$. Since $x''_i > 0$ and u_i is continuous in x_i, we have $u_i(z_\varepsilon, \theta'_i) > u_i(s, \theta'_i)$ for ε sufficiently low. The rest of the proof is the same as that of part (i). ◁

Let $\hat{M}_i = \{m_i \mid m_i = \mu_i(\theta_i) \text{ for some } \theta_i, \text{ and } q_i(m_i) > 0\}$. Lemma 7 implies that $|\hat{M}_i| \leq 2$, and if $|\hat{M}_i| = 2$, then there exists an $\hat{m}_i \in \hat{M}_i$ such that $\{\theta_i \mid \mu_i(\theta_i) = \hat{m}_i\}$ is a singleton.

We next show that if almost all types of legislator i send messages that result in a positive probability of inclusion or if almost all types of legislator i send messages that result in zero probability of inclusion, then legislator i is uninformative in (μ, γ, π).

Claim 5. *If* $P_i(q_i(\mu_i(\theta_i)) > 0) = 1$ *or* $P_i(q_i(\mu_i(\theta_i)) = 0) = 1$, *then legislator i is uninformative in* (μ, γ, π).

Proof. Suppose $P_i(q_i(\mu_i(\theta_i)) > 0) = 1$. Lemma 7 implies that there exists a message m^c_i such that $q_i(m^c_i) > 0$ and $P_i(\mu_i(\theta_i) = m^c_i) = 1$. Hence μ_i is equivalent to the size-1 message rule $\mu^{\mathrm{I}}_i(\theta_i) = m^c_i$ for all θ_i. Next, suppose $P_i(q_i(\mu_i(\theta_i)) = 0) = 1$. Consider a type $\hat{\theta}_i$ such that $q_i(\mu_i(\hat{\theta}_i)) = 0$ and a size-1 message rule $\mu^{\mathrm{I}}_i(\theta_i)$ such that $\mu^{\mathrm{I}}_i(\theta_i) = \mu_i(\hat{\theta}_i)$ for all θ_i. To see that μ_i is equivalent to μ^{I}_i, consider any θ_i such that $q_i(\mu_i(\theta_i)) = 0$. Note that in an SCE, for any m_j, $\pi(\mu_i(\hat{\theta}_i), m_j) = \pi(\mu_i(\theta_i), m_j)$ if both of these proposals exclude legislator i. Since $q_i(\mu_i(\hat{\theta}_i)) = q_i(\mu_i(\theta_i)) = 0$, it follows that $\pi(\mu_i(\hat{\theta}_i), \mu_j(\theta_j)) = \pi(\mu_i(\theta_i), \mu_j(\theta_j))$ for almost all $\theta_j \in \Theta_j$. Since $P_i(q_i(\mu_i(\theta_i)) = 0) = 1$, it follows that μ_i is equivalent to μ^{I}_i. ◁

Hence, if legislator i is informative in (μ, γ, π), then we have $P_i(q_i(\mu_i(\theta_i)) = 0) \in (0, 1)$ and $P_i(q_i(\mu_i(\theta_i)) > 0) \in (0, 1)$. By Lemma 7, if a positive measure of types send m_i that results in a positive probability of inclusion, then the types who send m_i form an interval at the lower end of Θ_i. Hence, if legislator i is informative in (μ, γ, π), then there exists a message m_i^c and a type $\theta_i^* \in (\underline{\theta}_i, \overline{\theta}_i)$ such that $q_i(m_i^c) > 0$ and $\mu_i(\theta_i) = m_i^c$ for all $\theta_i < \theta_i^*$, and $q_i(\mu_i(\theta_i)) = 0$ for almost all $\theta_i \geq \theta_i^*$. Pick any $\hat{\theta}_i$ such that $q_i(\mu_i(\hat{\theta}_i)) = 0$, and let $m_i^f = \mu_i(\hat{\theta}_i)$. Then μ_i is equivalent to μ_i^{II} such that $\mu_i^{\text{II}}(\theta_i) = m_i^c$ for $\theta_i < \theta_i^*$ and $\mu_i^{\text{II}}(\theta_i) = m_i^f$ for $\theta_i > \theta_i^*$. □

Proof of Proposition 4. Let $\Theta_i^0 = \{\theta_i \in \Theta_i \mid q_i(\mu_i(\theta_i)) = 0\}$. Proposition 3 and Lemma 7 imply that if legislator i is informative, then there exists $m_i^c \in M_i$ such that $q_i(m_i^c) > 0$, $P_i(\theta_i|\mu_i(\theta_i) = m_i^c) \in (0, 1)$, and $P_i(\theta_i|\mu_i(\theta_i) = m_i^c) + P_i(\Theta_i^0) = 1$. Let $\Theta_i^c = \{\theta_i \in \Theta_i \mid \mu_i(\theta_i) = m_i^c\}$.

Part (i). Suppose, to the contrary, that both legislators 1 and 2 are informative in (μ, γ, π). By Lemma 3, the chair's proposal in response to (m_1^c, m_2^c) excludes at least one legislator. To give an outline of the proof, we first show that if the chair's proposal in response to (m_1^c, m_2^c) excludes legislator 1, then by sending m_2^c, legislator 2 is included with probability 1 and any type of legislator 2 has an incentive to send m_2^c, a contradiction. We then show that if the chair's proposal in response to (m_1^c, m_2^c) excludes legislator 2, then by a similar argument, any type of legislator 1 has an incentive to send m_1^c, again a contradiction. We first establish the following claim.

Claim 6. *If $\theta_j \in \Theta_j^0$, then $\pi(m_i^c, \mu_j(\theta_j))$ excludes j.*

Proof. Suppose to the contrary that $\pi(m_i^c, \mu_j(\theta_j))$ includes j. Since $P_i(\Theta_i^c) > 0$, we have $q_j(\mu_j(\theta_j)) > 0$, a contradiction. ◁

Consider the following two cases.

(a) Suppose $\pi(m_1^c, m_2^c)$ excludes legislator 1. Consider any $\theta_1' \in \Theta_1^0$. By Claim 6, $\pi(\mu_1(\theta_1'), m_2^c)$ excludes legislator 1. Thus, in an SCE, $\pi(m_1^c, m_2^c) = \pi(\mu_1(\theta_1'), m_2^c)$ and, therefore, $\pi(\mu_1(\theta_1), m_2^c)$ is constant in θ_1 for $\theta_1 \in \Theta_1^c \cup \Theta_1^0$. Since $P_1(\Theta_1^c \cup \Theta_1^0) = 1$, this implies that $q_2(m_2^c)$ is either 0 or 1. Since $q_2(m_2^c) > 0$, we have $q_2(m_2^c) = 1$.

Since $\pi(m_1^c, m_2^c)$ excludes legislator 1 and $q_1(m_1^c) > 0$, we have that $P_2(\theta_2 \in \Theta_2^0 \mid \pi(m_1^c, \mu_2(\theta_2))$ includes 1$) > 0$. By Claim 6, the proposal $\pi(m_1^c, \mu_2(\theta_2))$ excludes 2 for all $\theta_2 \in \Theta_2^0$, and, therefore, in an SCE, $\pi(m_1^c, \mu_2(\theta_2))$ is constant in θ_2 for all $\theta_2 \in \Theta_2^0$. Hence $\pi(m_1^c, \mu_2(\theta_2))$ includes legislator 1 for all $\theta_2 \in \Theta_2^0$.

Recall that $V_i(m_i, \theta_i)$ is type θ_i's expected payoff from sending m_i in (μ, γ, π) (see page 488). Consider any type $\theta_2 \in \Theta_2^0$. Since $q_2(m_2^c) = 1$, Lemmas 3 and 5 imply that legislator 2 is pivotal with probability 1 when sending m_2^c and, therefore, $V_2(m_2^c, \theta_2) \geq u_2(s, \theta_2)$. Since $q_2(\mu_2(\theta_2)) = 0$, legislator 2 is excluded with probability 1 when sending $\mu_2(\theta_2)$. Moreover, since $\pi(m_1^c, \mu_2(\theta_2))$ includes legislator 1 and $P_1(\Theta_1^c) > 0$, Lemmas 3 and 5 imply that $V_2(\mu_2(\theta_2), \theta_2) < u_2(s, \theta_2)$. Hence any type $\theta_2 \in \Theta_2^0$ is strictly better off by sending m_2^c, a contradiction.

(b) Suppose $\pi(m_1^c, m_2^c)$ excludes legislator 2. Switching the roles of legislators 1 and 2, we can apply the same arguments as in case (a) to show that $q_1(m_1^c) = 1$ and any type $\theta_1 \in \Theta_1^0$ is strictly better off by sending m_1^c than $\mu_1(\theta_1)$, a contradiction.

Part (ii). Suppose, to the contrary, that legislator 2 is informative. Fix any type $\theta_2 \in \Theta_2^0$ and let $\Theta_1' = \{\theta_1 \mid \text{both } \pi(\mu_1(\theta_1), m_2^c) \text{ and } \pi(\mu_1(\theta_1), \mu_2(\theta_2)) \text{ exclude } 2\}$ and $\Theta_1'' = \{\theta_1 \mid \pi(\mu_1(\theta_1), m_2^c) \text{ includes 2 and } \pi(\mu_1(\theta_1), \mu_2(\theta_2)) \text{ excludes } 2\}$. If $\theta_1 \in \Theta_1'$, then $\pi(\mu_1(\theta_1), \mu_2(\theta_2)) = \pi(\mu_1(\theta_1), m_2^c)$ in an SCE. By Lemma 5, conditional on $\theta_1 \in \Theta_1''$, type θ_2's payoff is weakly higher than $u_2(s, \theta_2)$ if he sends m_2^c since $\pi(\mu_1(\theta_1), m_2^c)$ includes legislator 2. Since $e(\hat{y}_1) < e(\hat{y}_2)$, by Lemmas 4 and 5, conditional on $\theta_1 \in \Theta_1''$, type θ_2's payoff is strictly lower than $u_2(s, \theta_2)$ if he sends $\mu_2(\theta_2)$. Thus, if $P_1(\Theta_1' \cup \Theta_1'') = 1$ and $P_1(\Theta_1'') > 0$, then any type $\theta_2 \in \Theta_2^0$ is strictly better off by sending m_2^c, contradicting that (μ, γ, π) is an equilibrium. The rest of the proof shows that $P_1(\Theta_1' \cup \Theta_1'') = 1$ and $P_1(\Theta_1'') > 0$. Since $\theta_2 \in \Theta_2^0$, we have $q_2(\mu_2(\theta_2)) = 0$. Since $P_1(\Theta_1' \cup \Theta_1'') = P_1(\theta_1 | \pi(\mu_1(\theta_1), \mu_2(\theta_2)) \text{ excludes } 2) = 1 - q_2(\mu_2(\theta_2))$, it follows that $P_1(\Theta_1' \cup \Theta_1'') = 1$. By the definition of $q_2(\cdot)$, we also have $q_2(m_2^c) = P_1(\Theta_1'')$ and, therefore, $P_1(\Theta_1'') > 0$. □

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