Supplement to "The Formation of Networks with Local Spillovers and Limited Observability"

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C. ATTACHMENT KERNEL AND LINK INCENTIVE FUNCTION

Let $\mathcal{R}_t \subseteq \mathcal{S}_t$, $|\mathcal{R}_t| = m$, be the set of agents that receive a link from the entrant at time t. The network at time t is then given by $G_t = \langle \mathcal{P}_{t-1} \cup \{t\}, \mathcal{E}_{t-1} \cup \{tj : j \in \mathcal{R}_t\}\rangle$. We define the *attachment kernel* as the probability that an agent $j \in \mathcal{P}_{t-1}$ receives a link from the entrant

$$
\begin{aligned}
K_t^\beta(j|G_{t-1}) \equiv \mathbb{E}_t[\mathbb{1}_{\mathcal{R}_t}(j)|G_{t-1}] &= \sum_{\mathcal{S}_t \subseteq \mathcal{P}_{t-1}} \sum_{\mathcal{R}_t \subseteq \mathcal{S}_t} \mathbb{1}_{\mathcal{R}_t}(j) \mathbb{P}_t(\mathcal{S}_t, \mathcal{R}_t | G_{t-1}) \\
&= \sum_{\mathcal{S}_t \subseteq \mathcal{P}_{t-1}} \underbrace{\sum_{\mathcal{R}_t \subseteq \mathcal{S}_t} \mathbb{1}_{\mathcal{R}_t}(j) \mathbb{P}_t(\mathcal{R}_t | \mathcal{S}_t, G_{t-1})}_{\equiv K_t^\beta(j|\mathcal{S}_t, G_{t-1})} \mathbb{P}_t(\mathcal{S}_t|G_{t-1}),
\end{aligned}
$$

where $K_t^\beta(j|\mathcal{S}_t, G_{t-1})$ is the probability, conditional on the sample $\mathcal{S}_t$ and the prevailing network G_{t-1}, that an agent j receives a link after the m draws (without replacement) by the entrant, and β is a parameter related to the distribution of the additive error term ε_{tj} from Equation (2.2) (see below). Since the entrant forms links to the agents that maximize his link incentive function plus a random element, we need to consider the cases where agent j has the highest value among all agents in the sample, or the second highest, and so on. The corresponding probability can be written as follows[1]

(C.1)
$$
\begin{aligned}
K_t^\beta(j|\mathcal{S}_t, G_{t-1}) = \sum_{l=1}^{m} \sum_{i_1, i_2, \ldots, i_{l-1}} \prod_{r=1}^{l-1} \mathbb{P}_t \left(f_t^\delta(G_{t-1}, i_r) + \varepsilon_{t,i_r} = \max_{k \in \mathcal{S}_t \backslash \{i_1, \ldots, i_r\}} f_t^\delta(G_{t-1}, k) + \varepsilon_{t,k} \right) \\
\times \mathbb{P}_t \left(f_t^\delta(G_{t-1}, j) + \varepsilon_{t,j} = \max_{k \in \mathcal{S}_t \backslash \{i_1, \ldots, i_{l-1}\}} f_t^\delta(G_{t-1}, k) + \varepsilon_{t,k} \right) \mathbb{1}_{\mathcal{S}_t}(j),
\end{aligned}
$$

with indices $i_1 \in \mathcal{S}_t \backslash \{j\}$, $i_2 \in \mathcal{S}_t \backslash \{j, i_1\}$, $i_3 \in \mathcal{S}_t \backslash \{j, i_1, i_2\}$, $\ldots$, $i_{l-1} \in \mathcal{S}_t \backslash \{j, i_1, i_2, \ldots, i_{l-2}\}$ and $1 \leq l \leq m$. In the following I assume that the exogenous random terms ε_{tj} are identically and independently type I extreme value distributed (or Gumbel distributed) with parameter

[1] I assume that the entrant does not update the link incentive functions while forming links but evaluates it only once after he has observed the sample. The first sum in Equation (C.1) considers the case that agent j receives a link in the l-th round while the second sum takes into account all possible sequences of agents $i_1, i_2, \ldots, i_{l-1}$ that receive a link in the $l-1$ previous rounds.

η.[2] This assumption is commonly made in random utility models in econometrics (see e.g. McFadden, 1981). Under this distributional assumption, the probability that an entering agent t chooses the passive agent $j \in \mathcal{S}_t$ for creating the link tj (in the first of the m draws of link creation) follows a multinomial logit distribution given by (cf. Anderson et al., 1992)[3]

$$\begin{aligned}
\mathbb{P}_t\left(f_t^\delta(G_{t-1},j)+\varepsilon_{tj}=\max_{k\in\mathcal{S}_t} f_t^\delta(G_{t-1},k)+\varepsilon_{tk}\right) &= \frac{e^{\eta f_t^\delta(G_{t-1},j)}}{\sum_{k\in\mathcal{S}_t} e^{\eta f_t^\delta(G_{t-1},k)}} \\
&= \frac{1}{\sum_{k\in\mathcal{S}_t} e^{-\eta(f_t^\delta(G_{t-1},j)-f_t^\delta(G_{t-1},k))}} \\
&= \frac{1}{\sum_{k\in\mathcal{S}_t} e^{-\eta\delta^b(d_{G_{t-1}}(j)-d_{G_{t-1}}(k))+o(\delta^b)}} \\
&\approx \frac{e^{\beta d_{G_{t-1}}(j)}}{\sum_{k\in\mathcal{S}_t} e^{\beta d_{G_{t-1}}(k)}},
\end{aligned}$$

where we have applied condition (LD) for the link incentive function $f_t^\delta(G_{t-1},\cdot)$, dropped terms of the order $o(\delta^b)$ and denoted by $\beta \equiv \eta\delta^b$.

D. LARGE OBSERVATION RADIUS

D.1. *Sampling of Agents*

In the following we provide a lower bound on the observation radius n_s such that with high probability all agents in the network are observed by an entrant. Note that the probability that an agent $i \in \mathcal{P}_{t-1}$ does not enter the sample $\mathcal{S}_t$ is given by

$$\begin{aligned}
\mathbb{P}_t(i \notin \mathcal{S}_t|G_{t-1}) &= \left(1-\frac{1+d^-_{G_{t-1}}(i)}{t-1}\right)\left(1-\frac{1+d^-_{G_{t-1}}(i)}{t-2}\right)\cdots\left(1-\frac{1+d^-_{G_{t-1}}(i)}{t-1-(n_s-1)}\right) \\
&= \left(1-\frac{1+d^-_{G_{t-1}}(i)}{t}\right)^{n_s} + o\left(\frac{1}{t}\right) \qquad \text{(D.1)}
\end{aligned}$$

To see that this equality holds, note that when denoting by $c \equiv 1+d^-_{G_{t-1}}(i)$ we can write the above product as follows

$$\left(1-\frac{c}{t-1}\right)\left(1-\frac{c}{t-2}\right)\cdots\left(1-\frac{c}{t-1-(n_s-1)}\right) = \prod_{s=1}^{n_s}\left(1-\frac{c}{t-s}\right).$$

[2]The cumulative distribution function is given by $\mathbb{P}(\varepsilon \leq c) = \exp(-\exp(-\eta c-\gamma))$, where $\gamma \approx 0.577$ is Euler's constant. Mean and variance are given by $\mathbb{E}[\varepsilon]=0$ and $\mathrm{Var}(\varepsilon)=\frac{\pi^2}{6\eta^2}$.

[3] Assuming instead that we have a multiplicative error term ε_{tk} which follows an inverse exponential distribution with parameter η one can show that this probability can be written as $\mathbb{P}_t\left(f_t^\delta(G_{t-1},j)\cdot\varepsilon_{tj} = \max_{k\in\mathcal{S}_t} f_t^\delta(G_{t-1},k)\cdot\varepsilon_{tk}\right) = \frac{f_t^\delta(G_{t-1},j)^\eta}{\sum_{k\in\mathcal{S}_t} f_t^\delta(G_{t-1},k)^\eta}$, which corresponds to the ratio form of the contest success function (Jia, 2008).

Further, note that

$$\text{(D.2)} \qquad \left(1-\frac{c}{t-n_s}\right)^{n_s} \leq \prod_{s=1}^{n_s}\left(1-\frac{c}{t-s}\right) \leq \left(1-\frac{c}{t}\right)^{n_s}.$$

Now we have that

$$\frac{\left(1-\frac{c}{t}\right)^{n_s}}{\left(1-\frac{c}{t-n_s}\right)^{n_s}} = \left(\frac{(t-c)(t-n_s)}{t(t-c-n_s)}\right)^{n_s},$$

and using the fact that

$$\lim_{t\to\infty}\frac{(t-c)(t-n_s)}{t(t-c-n_s)} = 1,$$

it follows that both the lower and upper bound in Equation (D.2) converge to the same limit as t becomes large. Hence, we can write

$$\prod_{s=1}^{n_s}\left(1-\frac{c}{t-s}\right) = \left(1-\frac{c}{t}\right)^{n_s} + o\left(\frac{1}{t}\right).$$

Applying Bonferroni's inequality and neglecting terms of the order $o\left(\frac{1}{t}\right)$ in Equation (D.1), we then find that the probability that at least one of the agents in the set $\mathcal{P}_{t-1}$ is not observed by the entrant is bounded by $\mathbb{P}_t(\bigcup_{i\in\mathcal{P}_{t-1}}\{i \notin \mathcal{S}_t\}|G_{t-1}) \leq \sum_{i=1}^{t-1}\mathbb{P}_t(i \notin \mathcal{S}_t|G_{t-1}) \approx \sum_{k=0}^{t-2}\left(1-\frac{1+k}{t}\right)^{n_s}P_t(k) \approx \sum_{k=0}^{t-2}\left(1-n_s\frac{1+k}{t}\right)P_t(k) = 1-n_s\frac{1+m}{t}$, where we have assumed that $k=o_p(t)$, and used the fact that the average in-degree $\sum_{k=0}^{t-2}kP_t(k)$ equals the out-degree m. Hence, if we require the probability of an agent not being sampled to be lower than $\epsilon > 0$, then we must have that $n_s > t\frac{1-\epsilon}{1+m}$.

D.2. *Attachment Kernel*

The probability that an agent j with in-degree $d^-_{G_{t-1}}(j)$ receives a link in the $(k+1)$-st draw, given that the agents $l_1,\ldots,l_k$ have received a link in the previous k draws, $1\leq k\leq m$, is (cf. Equation (2.3))

$$\frac{e^{\beta d^-_{G_{t-1}}(j)}}{\sum_{i\in\mathcal{P}_{t-1}\backslash\{l_1,\ldots,l_k\}}e^{\beta d^-_{G_{t-1}}(i)}} \approx \frac{1+\beta d^-_{G_{t-1}}(j)}{\sum_{i\in\mathcal{P}_{t-1}\backslash\{l_1,\ldots,l_k\}}(1+\beta d^-_{G_{t-1}}(i))} = \frac{1+\beta d^-_{G_{t-1}}(j)}{(1+\beta m)t}\left(1+O\left(\frac{1}{t}\right)\right),$$

where we have used the approximation $e^{\beta x}\approx 1+\beta x$, and assumed that $d^-_{G_{t-1}}(i)=o_p(t)$ for all $i\in\mathcal{P}_{t-1}$. Moreover, we have used the fact that at every step t every passive agent has out-degree equal to m. Since the average out-degree must be equal to the average in-degree, we see that also the average in-degree must be m, and so $\sum_{i\in\mathcal{P}_{t-1}}(1+\beta d_{G_{t-1}}(i)) = (1+\beta m)t$. This probability is the same whether we use the in-degree $d^-_{G_{t-1}}(j)$ or the total degree $d_{G_{t-1}}(j)$, since they are related as $d_{G_{t-1}}(j) = d^+_{G_{t-1}}(j) + d^-_{G_{t-1}}(j) = m + d^-_{G_{t-1}}(j)$.

E. PAYOFF FUNCTIONS

This appendix contains a discussion of various models in the economic literature that satisfy Assumptions 1 and 2 introduced in Section 2.1.[4]

E.1. *Information Diffusion in Networks*

Following Fafchamps et al. (2010) I consider agents that exchange information in a network G, where information that travels longer paths is discounted by a factor $\delta \in [0, 1]$. It is assumed that information can travel both ways of a link and so I consider the (undirected) paths in the closure $\overline{G}$ of G. The probability that an agent j transmits information along a given path in $\overline{G}$ is independent of the probability that the same agent j transmits the same information along another path. With this assumption, the probability that agent i receives the information over all distances $k \geq 1$, when there are $c^k_{ij}(\overline{G})$ (undirected) paths of length k connecting i to j, becomes

$$P^\delta_{ij}(G) \equiv 1 - \prod_{k=1}^{\infty} (1 - \delta^k)^{c^k_{ij}(\overline{G})}.$$

The payoff $\pi_i : \mathcal{G}(n) \times \mathbb{R}_+ \to \mathbb{R}$ of agent i is defined as $\pi_i(G, \delta) \equiv V \sum_{j \in \mathcal{N}} P^\delta_{ij}(G) - c d^+_G(i)$ with $V > 0$ and a fixed cost $c \in [0, V\delta)$ for each link the agent has initiated. When the decay parameter δ is sufficiently small, we can write $(1 - \delta^k)^c \approx 1 - c\delta^k$. With this approximation the payoff of agent i becomes

$$\begin{aligned}
\pi_i(G, \delta) &\equiv V \sum_{j \in \mathcal{N}} \left(1 - \prod_{k=1}^{\infty} (1 - \delta^k)^{c^k_{ij}(\overline{G})} \right) - c d^+_G(i) \\
&= V \sum_{j \in \mathcal{N}} \left(1 - (1 - c^1_{ij}\delta)(1 - c^2_{ij}\delta^2) \right) + O(\delta^3) - c d_G(i) \\
&= V \sum_{j \in \mathcal{N}} \left(1 - 1 + c^1_{ij}\delta + c^2_{ij}\delta^2 - c^1_{ij}c^2_{ij}\delta^3 \right) + O(\delta^3) - c d_G(i) \\
&= V \left(\delta d_G(i) + \delta^2 \sum_{j \in \mathcal{N}_G(i)} d_G(j) \right) - c d^+_G(i) + O(\delta^3).
\end{aligned}$$

It then follows that the link incentive function is given by $f^\delta_i(G, j) = V\delta - c + V\delta^2 d_G(j) + O(\delta^3)$. Link monotonicity (LM) holds if $c < V\delta$ and linear differences (LD) holds for $g(x) = Vx$ and $\gamma = 2$, since $f^\delta_i(G, j) - f^\delta_i(G, k) = V\delta^2(d_G(j) - d_G(k)) + O(\delta^3)$. As our measure of

[4] All the models discussed here (which fall into our general framework) exhibit the property that the payoff of an agent is increasing with the number of collaborations, i.e. his degree. This characteristic has been found in empirical studies of coauthorship networks (e.g. Abbasi et al., 2011, Ductor, 2014).

welfare we consider aggregate payoff given by

$$
\begin{aligned}
\Pi(G,\delta) &= V\delta\sum_{i\in\mathcal{N}} d_G(i) + V\delta^2\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}_G(i)} d_G(j) + O(\delta^3) - c\sum_{i\in\mathcal{N}} d_G^+(i) \\
&= (2V\delta - c)e(\overline{G}) + V\delta^2\sum_{i\in\mathcal{N}} d_G(i)^2 + O(\delta^3) \\
&= (2V\delta - c)e(\overline{G}) + \frac{4V\delta^2}{n}e(\overline{G})^2 + V\delta^2 n\sigma_d^2(G) + O(\delta^3),
\end{aligned}
$$

where we have used the fact that $\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}_G(i)} d_G(j) = \sum_{i\in\mathcal{N}} d_G(i)^2$. The average degree is $\bar{d} = \frac{1}{n}\sum_{i=1}^n d_G(i) = \frac{2e(\overline{G})}{n}$. The degree variance is given by $\sigma_d^2(G) = \frac{1}{n}\sum_{i\in\mathcal{N}}(d_G(i) - \bar{d}_G) = \frac{1}{n}\sum_{i=1}^n d_G(i)^2 - \bar{d}^2 = \frac{1}{n}\sum_{i=1}^n d_G(i)^2 - \frac{4e(\overline{G})^2}{n^2}$. It follows that for small δ, such that terms of the order $O(\delta^3)$ become negligible, maximizing aggregate payoff $\Pi(G,\delta)$ (given n and e) becomes equivalent to maximizing the degree variance $\sigma_d^2(G)$, and condition (DC) holds.

E.2. *Two-Way Flow Communication*

The two-way flow model with decay has been introduced by Bala and Goyal (2000). In this model links are interpreted as lines of communication between two individuals. If i wants to communicate with j then i must first pay a fee of $c \geq 0$ to open the channel. By creating this link i does not only get access to j but also to all individuals that are approachable by j via an (undirected) path in the closure $\overline{G}$. Formally, the payoff function $\pi_i : \mathcal{G}(n) \times \mathbb{R}_+ \to \mathbb{R}$ of agent $i \in \mathcal{N}$ is given by[5]

$$
\pi_i(G,\delta) \equiv 1 + \sum_{i\neq j}\delta^{\ell(i,j,\overline{G})} - c d_G^+(i), \tag{E.1}
$$

for some $\delta \in [0,1]$, which is interpreted as the *degree of friction* in communication. The number $\ell(i,j,\overline{G})$ is the length of the shortest path connecting agent i with j in the graph $\overline{G}$. If i and j are not connected we adopt the convention that $\ell(i,j,\overline{G}) = \infty$. The difference to the payoff function in Fafchamps et al. (2010) of the previous section and the one in Equation (E.1) is that in the latter only the shortest paths matter.

In the following we assume that the network $\overline{G}$ does not contain any cycles, i.e. it is a tree (or a forest, if the network is unconnected). Denote by $\mathcal{T}(\mathcal{N})$ the class of (undirected) tree graphs with vertex set $\mathcal{N}$. Then a tree $\overline{G} \in \mathcal{T}(\mathcal{N})$ is defined by the conditions (i) that it is connected, and (ii) $|\mathcal{E}(\overline{G})| = |\mathcal{N}| - 1$ for all $\overline{G} \in \mathcal{T}(\mathcal{N})$. When $\overline{G} \in \mathcal{T}(\mathcal{N})$, the payoff of an agent $i \in \mathcal{N}$ can be written as

$$
\pi_i(G,\delta) = 1 + \delta d_G(i) + \delta^2\sum_{j\in\mathcal{N}_G(i)}(d_G(j) - 1) + O(\delta^3) - c d_G^+(i).
$$

[5] See also Jackson and Wolinsky (1996) for a similar payoff structure.

It follows that the linking incentive function of agent i takes the form

$$f_i^\delta(G,j) = \delta(1-\delta) - c + \delta^2 d_G(j) + O(\delta^3).$$

The link incentive function satisfies condition (LM) for $\delta(1-\delta) > c$ and condition (LD) with $g(x) = x$ and $\gamma = 2$, because $f_i^\delta(G,j) - f_i^\delta(G,k) = \delta^2(d_G(j) - d_G(k)) + O(\delta^3)$. Aggregate payoff $\Pi(G,\delta) = \sum_{i\in\mathcal{N}} \pi_i(G,\delta)$ is then given by

$$\begin{aligned}\Pi(G,\delta) &= n + \delta(1-\delta)\sum_{i\in\mathcal{N}} d_G(i) + \delta^2 \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}_G(i)} d_G(j) + O(\delta^3) - c\sum_{i\in\mathcal{N}} d_G^+(i) \\ &= n + (2\delta(1-\delta) - c)(n-1) + \frac{4\delta^2}{n}(n-1)^2 + n\delta^2\sigma_d^2(G) + O(\delta^3),\end{aligned}$$

where $e(\overline{G})$ is the number of edges in $\overline{G}$, $n = |\mathcal{N}|$, and we have used the fact that for $\overline{G} \in \mathcal{T}(\mathcal{N})$ the number of edges is $e(\overline{G}) = n-1$. It follows that for small δ such that terms of the order $O(\delta^3)$ become negligible, maximizing aggregate payoffs becomes equivalent to maximizing the degree variance . Hence, Condition (DC) holds for aggregate payoff when $\overline{G} \in \mathcal{T}[\mathcal{N}]$.[6]

E.3. *Public Goods Provision*

The following network game is presented in Goyal and Joshi (2006) as an extension of Bloch (1997). An (undirected) link between two agents represents an agreement to share knowledge about the production of a public good. Each agent can decide how much to invest into the public good. Denote the level of contribution of agent $i \in \mathcal{N} = \{1,\ldots,n\}$ as $x_i \in \mathbb{R}_+$. The production technology of every agent is assumed to be $c_i(x_i,G) = \frac{1}{2}\left(\frac{x_i}{d_G(i)+1}\right)^2$. The payoff function $\pi_i : \mathbb{R}_+^n \times \mathcal{G}(n) \to \mathbb{R}$ of agent i is

$$\pi_i(\mathbf{x},G) \equiv \sum_{j\in\mathcal{N}} x_j - \frac{1}{2}\left(\frac{x_i}{d_G(i)+1}\right)^2.$$

The Nash contribution of agent i is $x_i^* = (d_G(i)+1)^2$. This optimal choice of an agent induces naturally preferences over networks by inserting the value of $x_i(G)$ into the payoff function π_i. This gives us

$$\pi_i(G) \equiv \pi_i(\mathbf{x}^*,G) = \frac{1}{2}\left(d_G(i)+1\right)^2 + \sum_{j\in\mathcal{N}\setminus\{i\}} (d_G(j)+1)^2.$$

[6]We will see in the network growth model introduced in Section 2.2 that $\overline{G} \in \mathcal{T}[\mathcal{N}]$ is always guaranteed to hold if we allow an entering agent to form only a single link.

With this payoff function, the linking incentive function for an agent i is given by

$$f_i^\delta(G,j) = \frac{9}{2} + 2d_G(j).$$

This obviously satisfies conditions (LM) and (LD) with $g(x) = 2x$ and $\gamma = 0$. Aggregate payoff $\Pi(G) = \sum_{i\in\mathcal{N}} \pi_i(G)$ is then given by

$$\begin{aligned}\Pi(G) &= \frac{1}{2}\sum_{i\in\mathcal{N}}(d_G(i)+1)^2 + \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}\setminus\{i\}}(d_G(j)+1)^2 \\ &= \frac{n(2n-1)}{2} + 2(2n-1)\left(1+\frac{\delta^2}{n}e(\overline{G})\right)e(\overline{G}) + \frac{n(2n-1)\delta^2}{2}\sigma_d^2(G).\end{aligned}$$

We see that aggregate payoffs are increasing in the degree variance and condition (DC) holds.

E.4. *A Linear-Quadratic Complementarity Game*

We consider a simplified form of the game introduced by Ballester et al. (2006) where each agent $i\in\mathcal{N}$ in the network G selects an effort level $x_i \geq 0$, $\mathbf{x}\in\mathbb{R}_+^n$ (e.g. the R&D investment of a firm or the working hours of an inventor), and receives a payoff $\pi_i : \mathbb{R}_+^n \times \mathcal{G}(n)\times\mathbb{R}_+ \to \mathbb{R}$ of the following form

$$\text{(E.2)} \qquad \pi_i(\mathbf{x},G,\delta) \equiv x_i - \frac{1}{2}x_i^2 + \delta\sum_{j=1}^n a_{ij}x_ix_j,$$

where $\delta \geq 0$ and $a_{ij}\in\{0,1\}$, $i,j\in\mathcal{N}=\{1,\ldots,n\}$ are the elements of the symmetric $n\times n$ *adjacency matrix* $\mathbf{A}$ of $\overline{G}$. This payoff function is additively separable in the idiosyncratic effort component ($x_i - \frac{1}{2}x_i^2$) and the peer effect contribution ($\delta\sum_{j=1}^n a_{ij}x_ix_j$). Payoffs display strategic complementarities in effort levels, i.e., $\frac{\partial^2\pi_i(\mathbf{x},G,\delta)}{\partial x_i\partial x_j} = \delta a_{ij} \geq 0$. Ballester et al. (2006) have shown that if $\delta < 1/\lambda_{\mathrm{PF}}(G)$ then the unique interior Nash equilibrium solution of the simultaneous n–player move game with payoffs given by Equation (E.2) and strategy space $\mathbb{R}_+^n$ is given by the Bonacich centrality $x_i^* = b_i(G,\delta)$ for all $i\in\mathcal{N}$ (Bonacich, 1987).[7] Moreover, the payoff of agent i in equilibrium is given by

$$\text{(E.3)} \qquad \pi_i(G,\delta) \equiv \pi_i(\mathbf{x}^*,G,\delta) = \frac{1}{2}(x_i^*)^2 = \frac{1}{2}b_i^2(G,\delta).$$

[7]Let $\lambda_{\mathrm{PF}}(G)$ be the largest real (Perron-Frobenius) eigenvalue of the adjacency matrix $\mathbf{A}$ of the undirected network $\overline{G}$. If $\mathbf{I}$ denotes the $n\times n$ identity matrix and $\mathbf{u}\equiv(1,\ldots,1)^\top$ the n-dimensional vector of ones then we can define the *Bonacich centrality* as follows: If and only if $\delta < 1/\lambda_{\mathrm{PF}}(G)$ then the matrix $\mathbf{B}(G,\delta) \equiv (\mathbf{I}-\delta\mathbf{A})^{-1} = \sum_{k=0}^\infty \delta^k\mathbf{A}^k$ exists, is non-negative (see e.g. Debreu and Herstein, 1953), and the vector of Bonacich centralities is defined as $\mathbf{b}(G,\delta) \equiv \mathbf{B}(G,\delta)\cdot\mathbf{u}$. We can write the vector of Bonacich centralities as $\mathbf{b}(G,\delta) = \sum_{k=0}^\infty \delta^k\mathbf{A}^k\cdot\mathbf{u} = (\mathbf{I}-\delta\mathbf{A})^{-1}\cdot\mathbf{u}$. For the components $b_i(G,\delta)$, $i=1,\ldots,n$, we get $b_i(G,\delta) = \sum_{k=0}^\infty \delta^k(\mathbf{A}^k\cdot\mathbf{u})_i = \sum_{k=0}^\infty \delta^k\sum_{j=1}^n\left(\mathbf{A}^k\right)_{ij}$, where $\left(\mathbf{A}^k\right)_{ij}$ is the ij-th entry of $\mathbf{A}^k$. Because $\sum_{j=1}^n\left(\mathbf{A}^k\right)_{ij}$ is the number of all (undirected) walks of length k in $\overline{G}$ starting from i, $b_i(G,\delta)$ is the number of all walks in $\overline{G}$ starting from i, where the walks of length k are weighted by their geometrically decaying factor δ^k.

In the case of small complementarity effects, corresponding to small values of δ, the Bonacich centrality of an agent i can be written as

$$b_i(G,\delta) = 1 + \delta d_G(i) + \delta^2 \sum_{j \in \mathcal{N}_G(i)} d_G(j) + O(\delta^3).$$

Note that equilibrium payoff can be written as

$$\pi_i(G,\delta) = \frac{1}{2} + \delta d_G(i) + \frac{\delta^2}{2} d_G(i)^2 + \delta^2 \sum_{j \in \mathcal{N}_G(i)} d_G(j) + O(\delta^3),$$

and the link incentive function is then given by

$$f_i^\delta(G,j) = \frac{\delta(2+\delta)}{2} + \frac{\delta^2}{2} d_G(i)(d_G(i)+1) + \delta^2 d_G(j) + O(\delta^3).$$

If we neglect terms of the order $O(\delta^3)$ then the linking incentive function also satisfies condition (LM). Further, $f_i^\delta(G,j) - f_i^\delta(G,k) = \delta^2(d_G(j) - d_G(k)) + O(\delta^3)$ so that condition (LD) holds with $g(x) = x$ and $\gamma = 2$. Aggregate payoff $\Pi(G,\delta) = \sum_{i \in \mathcal{N}} \pi_i(G,\delta)$ can be written as

$$\begin{aligned}
\Pi(G,\delta) &= \frac{n}{2} + \delta \sum_{i=1}^{n} d_G(i) + \frac{\delta^2}{2} \sum_{i=1}^{n} d_G(i)^2 + \delta^2 \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_G(i)} d_G(j) + O(\delta^3) \\
&= \frac{n}{2} + 2\delta \left(1 + \frac{3\delta}{n} e(\overline{G})\right) e(\overline{G}) + \frac{3n\delta^2}{2} \sigma_d^2(G) + O(\delta^3).
\end{aligned}$$

Aggregate payoff is increasing in the degree variance, and hence, condition (DC) holds.

F. The LF-MCMC Algorithm

The purpose of the likelihood-free Markov chain Monte Carlo (LF-MCMC) algorithm is to estimate the parameter vector $\boldsymbol{\theta} \equiv (\beta, p, n_s, m)_{1\times L}$, $L = 4$, of the model on the basis of the summary statistics $\mathbf{S} \equiv (\mathbf{S}_1, \ldots, \mathbf{S}_K)_{T\times K}$, $K = 4$, where $\mathbf{S}_1 \equiv (P(k))_{k=0}^{T-1}$, $\mathbf{S}_2 \equiv (C(k))_{k=0}^{T-1}$, $\mathbf{S}_3 \equiv (k_{\mathrm{nn}}(k))_{k=0}^{T-1}$ and $\mathbf{S}_4 \equiv (P(s))_{s=1}^{T}$. The algorithm generates a Markov chain which is a sequence of parameters $(\boldsymbol{\theta}_s)_{s=1}^{n}$ with a stationary distribution that approximates the distribution of each parameter value $\theta \in \boldsymbol{\theta}$ conditional on the observed statistic $\mathbf{S}^o$.

Definition 1 *Consider the statistics $\mathbf{S}$ and denote by $\mathbf{S}^o$ the observed statistics. Further, let $\Delta(\mathbf{S}_i^o, \mathbf{S}_i)$ be a measure of distance between the i-th realized statistic $\mathbf{S}_i$ of the network formation process $(G_t)_{t=1}^T$ with parameter vector $\boldsymbol{\theta}$ and the i-th observed statistic $\mathbf{S}_i^o$ for $i = 1, \ldots, K$. Then we consider the Markov chain $(\boldsymbol{\theta}_s)_{s=1}^n$ induced by the following algorithm:*

(i) *Given $\boldsymbol{\theta}$, propose $\boldsymbol{\theta}'$ according to the proposal density $q_s(\boldsymbol{\theta} \to \boldsymbol{\theta}')$.*

(ii) *Generate a network $G_T(\boldsymbol{\theta}')$ according to $\boldsymbol{\theta}'$ and calculate the summary statistics $\mathbf{S}'$.*

(iii) *Calculate*

$$h(\boldsymbol{\theta}, \boldsymbol{\theta}') = \min\left(1, \frac{q_s(\boldsymbol{\theta}' \to \boldsymbol{\theta})}{q_s(\boldsymbol{\theta} \to \boldsymbol{\theta}')} \prod_{i=1}^{K} \mathbb{1}_{\{\Delta(\mathbf{S}'_i, \mathbf{S}^o_i) < \epsilon_{i,s}\}}\right),$$

where $\epsilon_{i,s} \geq 0$ *is a monotonic decreasing sequence of threshold values,* $\epsilon_{i,s} \downarrow \epsilon_i^{\min}$*, and* $\Delta : \mathbb{R}_+^T \times \mathbb{R}_+^T \to \mathbb{R}_+$ *is a distance metric in* $\mathbb{R}_+^T$*.*

(iv) *Accept* $\boldsymbol{\theta}'$ *with probability* $h(\boldsymbol{\theta}, \boldsymbol{\theta}')$*, otherwise stay at* $\boldsymbol{\theta}$ *and go to (i).*

Marjoram et al. (2003) have shown that the distribution generated by the above algorithm converges to the true conditional distribution of the parameter vector $\boldsymbol{\theta}$, given the observations $\mathbf{S}^o$ and the threshold values. Their result is stated more formally in the following proposition.

PROPOSITION 1 *The stationary distribution* $f : \mathbb{R}^K \to [0,1]^K$ *of the Markov chain* $(\boldsymbol{\theta}_s)_{s=1}^n$ *is given by*

$$f\left(\boldsymbol{\theta} \middle| \prod_{i=1}^{K} \mathbb{1}_{\{\Delta(\mathbf{S}_i, \mathbf{S}^o_i) < \epsilon_i^{\min}\}}\right).$$

PROOF OF PROPOSITION 1: Let us denote the transition probability of the Markov chain $(\boldsymbol{\theta}_s)_{s=1}^n$ from state $\boldsymbol{\theta}$ to state $\boldsymbol{\theta}'$ by $p_s(\boldsymbol{\theta} \to \boldsymbol{\theta}')$. Assume w.l.o.g. that for $\boldsymbol{\theta} \neq \boldsymbol{\theta}'$ and $1 \leq s \leq n$ it holds that

$$\text{(F.1)} \qquad \frac{q_s(\boldsymbol{\theta}' \to \boldsymbol{\theta})}{q_s(\boldsymbol{\theta} \to \boldsymbol{\theta}')} \leq 1.$$

Consider the distribution of the parameter vector $\boldsymbol{\theta}$, conditional on the event $\{\Delta(\mathbf{S}^o, \mathbf{S}) \leq \epsilon\} \equiv \prod_{i=1}^{K} \mathbb{1}_{\{\Delta(\mathbf{S}_i, \mathbf{S}^o_i) < \epsilon_i^{\min}\}}$, that is $f(\boldsymbol{\theta}|\Delta(\mathbf{S}^o, \mathbf{S}) \leq \epsilon) = \mathbb{P}(\Delta(\mathbf{S}^o, \mathbf{S}) \leq \epsilon|\boldsymbol{\theta})/\mathbb{P}(\Delta(\mathbf{S}^o, \mathbf{S}) \leq \epsilon)$. We have that

$$\begin{aligned}
f(\boldsymbol{\theta}|\Delta(\mathbf{S}^o, \mathbf{S}) \leq \epsilon) p_s(\boldsymbol{\theta} \to \boldsymbol{\theta}') &= \frac{\mathbb{P}(\Delta(\mathbf{S}^o, \mathbf{S}) \leq \epsilon|\boldsymbol{\theta})}{\mathbb{P}(\Delta(\mathbf{S}^o, \mathbf{S}) \leq \epsilon)} \mathbb{P}(\Delta(\mathbf{S}^o, \mathbf{S}') \leq \epsilon|\boldsymbol{\theta}') q_s(\boldsymbol{\theta} \to \boldsymbol{\theta}') \frac{q_s(\boldsymbol{\theta}' \to \boldsymbol{\theta})}{q_s(\boldsymbol{\theta} \to \boldsymbol{\theta}')} \\
&= \frac{\mathbb{P}(\Delta(\mathbf{S}^o, \mathbf{S}') \leq \epsilon|\boldsymbol{\theta}')}{\mathbb{P}(\Delta(\mathbf{S}^o, \mathbf{S}) \leq \epsilon)} \mathbb{P}(\Delta(\mathbf{S}^o, \mathbf{S}) \leq \epsilon|\boldsymbol{\theta}) q_s(\boldsymbol{\theta}' \to \boldsymbol{\theta}) \\
&= f(\boldsymbol{\theta}'|\Delta(\mathbf{S}^o, \mathbf{S}') \leq \epsilon) q_s(\boldsymbol{\theta}' \to \boldsymbol{\theta}) \mathbb{P}(\Delta(\mathbf{S}^o, \mathbf{S}) \leq \epsilon|\boldsymbol{\theta}) h(\boldsymbol{\theta}', \boldsymbol{\theta}) \\
&= f(\boldsymbol{\theta}'|\Delta(\mathbf{S}^o, \mathbf{S}') \leq \epsilon) p_s(\boldsymbol{\theta}' \to \boldsymbol{\theta}),
\end{aligned}$$

where we have used the fact that $h(\boldsymbol{\theta}', \boldsymbol{\theta}) = 1$ if the inequality in (F.1) is satisfied. It follows that $f(\boldsymbol{\theta}|\Delta(\mathbf{S}^o, \mathbf{S}) \leq \epsilon)$ satisfies a detailed balance condition and therefore is the stationary distribution of the Markov chain. *Q.E.D.*

The proposal distribution $q_s(\boldsymbol{\theta} \to \boldsymbol{\theta}')$ is a truncated normal distribution $\boldsymbol{\theta}' \sim \mathcal{N}(\boldsymbol{\theta}, \boldsymbol{\Sigma}_s)$

$\mathbb{1}_{[\boldsymbol{\theta}^{\min},\boldsymbol{\theta}^{\max}]}(\boldsymbol{\theta})$ for each parameter $\theta \in \boldsymbol{\theta}$ with a diagonal variance-covariance matrix $\boldsymbol{\Sigma}_s = \mathrm{diag}\{\sigma_{1,s}^2, \ldots, \sigma_{L,s}^2\}$. More precisely, for each continuous parameter $\theta_i \in \mathbb{R}_+$ (i.e. p, β) I choose a proposal distribution given by

$$q_s(\theta_i \to \theta_i') = \frac{\phi(\theta'|\theta, \sigma_{i,s}^2)}{\Phi(\theta_i^{\max}|\theta_i, \sigma_{i,s}^2) - \Phi(\theta_i^{\min}|\theta_i, \sigma_{i,n}^2)} \mathbb{1}_{[\theta_i^{\min},\theta_i^{\max}]}(\theta_i'),$$

where $\phi(\theta|\mu, \sigma^2)$ and $\Phi(\theta|\mu, \sigma^2)$ are the pdf and cdf, respectively, of a normally distributed random variable with mean μ and variance σ^2. For the discrete parameters $\theta_i \in \mathbb{Z}_+$ (i.e. n_s, while m is set through the condition $\bar{d} = mp$ when the network is directed while $\bar{d} = 2pm$ when it is undirected), I choose a proposal distribution given by

$$q_s(\theta_i \to \theta_i') = \frac{\Phi(\theta_i' + 1|\theta, \sigma_{i,s}^2) - \Phi(\theta_i'|\theta, \sigma_{i,s}^2)}{\Phi(\theta_i^{\max}|\theta_i, \sigma_{i,s}^2) - \Phi(\theta_i^{\min}|\theta_i, \sigma_{i,s}^2)} \mathbb{1}_{[\theta_i^{\min},\theta_i^{\max}]}(\theta_i').$$

During the "burn-in" phase (Chib, 2001), I consider a monotonic decreasing sequence of thresholds given by $\epsilon_{i,s} \geq \epsilon_{i,s+1} \geq \ldots \geq \epsilon_i^{\min}$ with $\epsilon_{i,s+1} = \max\left\{(1-\gamma)\epsilon_{i,s}, \epsilon_i^{\min}\right\}$ and $\gamma = 0.05$. Similarly, I assume a decreasing sequence of variances $\sigma_{i,s}^2 \geq \sigma_{i,s+1}^2 \geq \ldots \geq (\sigma_i^{\min})^2$ with $\sigma_{i,s+1}^2 = \max\left\{(1-\gamma)\sigma_{i,s}^2, (\sigma_i^{\min})^2\right\}$ for the proposal distribution $q_s(\theta_i \to \theta_i')$. The maximum number of iterations, n, has been chosen such that reasonably high values of $p_\theta(n)$ were obtained. As a measure of distance I choose the Euclidean distance $\Delta(\mathbf{S}_i, \mathbf{S}_i^o) = \sqrt{\sum_{j=1}^T \left(S_{i,j} - S_{i,j}^o\right)^2}$. The parameter ranges are $n_s \in \{1, \ldots, 100\}$, $p \in [0, 1]$ and $\beta \in [0, 100]$. The parameters $\epsilon_i^{\min}$ are choose sufficiently small after long experimentation with different starting values and burn-in periods.

G. UNDIRECTED LINKS

In the following network formation process we allow entering agents to observe not only the out-neighbors of incumbent agents but also their in-neighbors. The resulting network can then be viewed as an undirected graph. The precise definition of the network growth process is given below:

DEFINITION 2 *For a fixed $T \in \mathbb{N} \cup \{\infty\}$ we define a network formation process $(G_t)_{t\in[T]}$ as follows. Given the initial graph $G_1 = \ldots = G_{m+1} = K_{m+1}$, for all $t > m+1$ the graph G_t is obtained from G_{t-1} by applying the following steps:*

Growth: *Given $\mathcal{P}_1$ and $\mathcal{A}_1$, for all $t \geq 2$ the agent sets in period t are given by $\mathcal{P}_t = \mathcal{P}_{t-1} \cup \{t\}$ and $\mathcal{A}_t = \mathcal{A}_{t-1} \setminus \{t\}$, respectively.*

Network sampling: *Agent t observes a sample $\mathcal{S}_t \subseteq \mathcal{P}_{t-1}$. The sample $\mathcal{S}_t$ is constructed by selecting without replacement $n_s \geq 1$ agents $i \in \mathcal{P}_{t-1}$ uniformly at random and adding i as well as the neighbors $\mathcal{N}_{G_{t-1}}(i)$ of i to $\mathcal{S}_t$.*

Link creation: *Given the sample $\mathcal{S}_t$, agent t creates $m \geq 1$ links to agents in $\mathcal{S}_t$ without replacement. For each link, agent t chooses the $j \in \mathcal{S}_t$ that maximizes $f_t^\delta(G_{t-1}, j) + \varepsilon_{tj}$.*

G.1. *Large Observation Radius*

We first consider the case of $\mathcal{S}_t = \mathcal{P}_{t-1}$. Let $k_j(t)$ denote the degree of agent j at time t. Considering only the leading terms in $O\left(\frac{1}{t}\right)$ we can write the probability that an agent $j \in \mathcal{P}_{t-1}$ to receive a link by the entrant t as follows

$$K_t^{\beta}(j|G_{t-1}) \approx \frac{m}{1+2\beta m}\frac{1+\beta d_{G_{t-1}}(j)}{t}. \tag{G.1}$$

Using the recursive Equation (B.3) with the attachment kernel in Equation (G.1) yields the following proposition.

Proposition 2 *Consider the sequence of degree distributions $\{P_t\}_{t\in\mathbb{N}}$ generated by an indefinite iteration of the network formation process $(G_t^{\beta})_{t\in\mathbb{N}}$ introduced in Definition 2 with n_s large enough such that $\mathcal{S}_t = \mathcal{P}_{t-1}$ for every $t > m+1$. Then, for all $k \geq 0$ we have in the limit $\beta \to 0$ that $P_t(k) \to P^{\beta}(k)$, where*

$$P^{\beta}(k) = \frac{(1+2m\beta)\Gamma\left(k+\frac{1}{\beta}\right)\Gamma\left(3+\frac{1}{\beta}+\frac{1}{m\beta}\right)}{(1+m+2m\beta)\Gamma\left(\frac{1}{\beta}\right)\Gamma\left(k+3+\frac{1}{\beta}+\frac{1}{m\beta}\right)}. \tag{G.2}$$

Proof of Proposition 2: Equation (G.2) follows directly from the recursion in Equation (B.3) and the attachment kernel in Equation (G.1). *Q.E.D.*

From Equation (G.2) we find that the large k behavior of the degree distribution follows a power-law as $P^{\beta}(k) \sim k^{-\left(3+\frac{1}{m\beta}\right)}$. In the continuum approximation we can write for the dynamics of $k_s(t)$ using Equation (G.1) as

$$\frac{dk_s(t)}{dt} = \frac{m}{1+2\beta m}\frac{1+\beta k_j(t)}{t},$$

with the initial condition $k_s(s) = m$. The solution is given by

$$k_s(t) = \frac{1}{\beta}\left((1+\beta m)\left(\frac{t}{s}\right)^{\frac{\beta m}{1+2\beta m}} - 1\right), \tag{G.3}$$

and we obtain for the degree distribution in the continuum approximation

$$P^{\beta}(k) = \frac{1+2\beta m}{m}(1+\beta m)^{2+\frac{1}{\beta m}}(1+\beta k)^{-\left(3+\frac{1}{m\beta}\right)}, \tag{G.4}$$

with $\int_0^{\infty} P^{\beta}(k)dk = 1$. This yields the same asymptotic behavior of the degree distribution as in Equation (G.2).

Next, we turn to the average nearest neighbor connectivity.

PROPOSITION 3 *Consider the network formation process* $(G_t^\beta)_{t\in\mathbb{R}_+}$ *of Definition 2 with* $\mathcal{S}_t = \mathcal{P}_{t-1}$ *for all* $t > m+1$ *in the continuum approximation and assume that Equation (G.3) holds. Then in the limit* $\beta \to 0$ *the nearest-neighbor degree distribution is given by*

$$k_{nn}(k) = \frac{1}{\beta^2 k}\left(1 + \frac{1+\beta k}{1+\beta m}\left(\beta^2 R_s(s) - 1 + (1+\beta m)^2 \ln\left(\frac{1+\beta k}{1+\beta m}\right)\right)\right), \tag{G.5}$$

where $a = \frac{m}{1+2\beta m}$, *the initial condition*

$$R_{s+1}(s+1) = \frac{a(1-\beta)(1-2m\beta)}{\beta} + \frac{a(1+\beta m)^2}{\beta} s^{2\beta a - 1} \sum_{j=1}^{s} \frac{1}{j^{2\beta a}},$$

and $s = t\left(\frac{1+\beta m}{1+\beta k}\right)^{2+\frac{1}{m\beta}}$.

Asymptotically, only the last term in Equation (G.5) is relevant and we obtain

$$k_{\mathrm{nn}}(k) \sim \frac{1+\beta m}{\beta} \ln\left(\frac{1+\beta k}{1+\beta m}\right), \tag{G.6}$$

as $k \to \infty$.

PROOF OF PROPOSITION 3: Denote by $R_s(t) = \sum_{j\in\mathcal{N}_{G_t}(s)} k_j(t)$ the sum of the degrees of the neighbors of vertex s at time t. We can write

$$\begin{aligned}\frac{dR_s(t)}{dt} &= \frac{m^2}{1+2\beta m}\frac{1+\beta k_s(t)}{t} + \sum_{j\in\mathcal{N}_{G_t}(s)} \frac{m}{1+2\beta m}\frac{1+\beta k_j(t)}{t} \\ &= \frac{a}{t}\left(m + (1+\beta m)k_s(t) + \beta R_s(t)\right) = \frac{a}{\beta t}\left((1+\beta m)^2\left(\frac{t}{s}\right)^{\beta a} + \beta^2 R_s(t)\right),\end{aligned}$$

where we have denoted by $a = \frac{m}{1+2\beta m}$ and using the fact that$1 + \beta k_s(t) = (1+\beta m)\left(\frac{t}{s}\right)^{\beta a}$ from Equation (G.3) under the continuum approximation. The initial condition is given by

$$R_s(s) = \sum_{j=1}^{s} \frac{a}{s}(1+\beta k_j(s))(1+k_j(s)) = \frac{a(1-\beta)(1-2m\beta)}{\beta} + \frac{a}{s}\sum_{j=1}^{s}(1+\beta k_j(s))^2.$$

Using the fact that

$$1 + \beta k_j(s) = (1+\beta m)\left(\frac{s}{j}\right)^{\beta a}, \tag{G.7}$$

we obtain

$$R_s(s) = \frac{a(1-\beta)(1-2m\beta)}{\beta} + \frac{a(1+\beta m)^2}{\beta} s^{2\beta a-1} H(s, 2\beta a).$$

We then get

(G.8)
$$R_s(t) = \frac{1}{\beta^2}\left(1 + \left(a\beta(1+\beta m)^2 \left(\frac{1}{s}H(s,2a\beta) + (1+m\beta)\ln\left(\frac{t}{s}\right)\right) - 1 + \beta^2 b\right)\left(\frac{t}{s}\right)^{a\beta}\right).$$

Using once again Equation (G.7) and inserting into $k_{\mathrm{nn}} = \frac{R_s}{k}$ delivers Equation (G.5). *Q.E.D.*

Moreover, we can compute the clustering degree distribution as provided in the next proposition.

PROPOSITION 4 *Consider the network formation process $(G_t^\beta)_{t\in\mathbb{R}_+}$ of Definition 2 with $\mathcal{S}_t = \mathcal{P}_{t-1}$ for all $t > m+1$ in the continuum approximation and assume that Equation (G.3) holds. Then in the limit $\beta \to 0$ the clustering degree distribution is given by*

$$C(k) = \frac{2}{k(k-1)}\left(M_s + \frac{b}{s(1-2a\beta)}\left(d + a\beta s^{2a\beta-1}\left(1-\left(\frac{t}{s}\right)^{2a\beta-1}\right)H_s^{2\beta a}\right.\right.$$
$$\left.\left. - \left(\frac{t}{s}\right)^{2a\beta-1}\left(d + \ln\left(\frac{t}{s}\right)^{a\beta}\right)\right)\right), \tag{G.9}$$

where $s = t\left(\frac{1+m\beta}{1+k\beta}\right)^{2+\frac{1}{m\beta}}$, $a = \frac{a}{1+2\beta m}$, $b = \frac{m(m-1)(1+\beta m)^2}{\beta(1+2\beta m)}$, $c = \frac{\beta m + a\beta(1-\beta)(1-2m\beta)}{(1+\beta m)^2}$, $d = \frac{c+a\beta(1-2c)}{1-2a\beta}$, the Harmonic number is defined as $H_s^a \equiv \sum_{j=1}^s j^{-a}$ and the initial condition is given by

$$M_{s+1}(s+1) = \frac{m(m-1)s^{2a-2}}{(1+2\beta m)^2}\left(\sum_{i=1}^m \frac{1}{i^a}\sum_{j=i+1}^m \frac{1}{j^a} + \frac{2m}{1+2\beta m}\sum_{i=m+1}^s \frac{1}{i^{2a}}\sum_{j=i}^{s-1}\frac{1}{j}\right).$$

The large k behavior of the clustering coefficient is dominated by the second term in Equation (G.9), yielding

(G.10)
$$C(k) \sim \frac{2bd}{k(k-1)s(1-2a\beta)} = \frac{1}{t}\frac{2bd}{(1-2a\beta)(1+m\beta)^{2+\frac{1}{m\beta}}}\frac{(1+\beta k)^{2+\frac{1}{m\beta}}}{k(k-1)} = O\left(\frac{1}{t}k^{\frac{1}{m\beta}}\right), \quad k \to \infty.$$

Proof of Proposition 4: Let $M_s(t)$ denote the number of triangles containing s at time t. We have that

$$\begin{aligned}\frac{dM_s(t)}{dt} &= \frac{m}{1+2\beta m}\frac{1+\beta k_s(t)}{t}\sum_{j\in\mathcal{N}_{G_t}(s)}\frac{m-1}{1+2\beta m}\frac{1+\beta k_j(t)}{t} \\ &= \frac{m(m-1)(1+\beta k_s(t))}{(1+2\beta m)^2 t^2}(k_s(t)+\beta R_s(t)).\end{aligned}$$

With $R_s(t)$ from Equation (G.8) and Equation (G.7) we obtain

$$\frac{dM_s(t)}{dt} = \frac{b}{t^2}\left(\frac{t}{s}\right)^{2\beta a}\left(c+\ln\left(\frac{t}{s}\right)^{\beta a}+a\beta(s)^{2\beta a-1}H_s^{2\beta a}\right),$$

where $a=\frac{a}{1+2\beta m}$, $b=\frac{m(m-1)(1+\beta m)^2}{\beta(1+2\beta m)}$, $c=\frac{\beta m+a\beta(1-\beta)(1-2m\beta)}{(1+\beta m)^2}$ and the Harmonic number is defined as $H_s^a\equiv\sum_{j=1}^s j^{-a}$. The solution is given by

$$M_s(t)=M_s(s)+\frac{b}{s(1-2a\beta)}\left(d+a\beta s^{2a\beta-1}\left(1-\left(\frac{t}{s}\right)^{2a\beta-1}\right)H_s^{2\beta a}-\left(\frac{t}{s}\right)^{2a\beta-1}\left(d+\ln\left(\frac{t}{s}\right)^{a\beta}\right)\right),$$

where $d=\frac{c+a\beta(1-2c)}{1-2a\beta}$. Similar to the derivation of Equation (B.24), the initial condition is given by

$$M_{s+1}(s+1)=\frac{m(m-1)s^{2a-2}}{(1+2\beta m)^2}\left(\sum_{i=1}^m\frac{1}{i^a}\sum_{j=i+1}^m\frac{1}{j^a}+\frac{2m}{1+2\beta m}\sum_{i=m+1}^s\frac{1}{i^{2a}}\sum_{j=i+1}^s\frac{1}{j-1}\right).$$

Using Equation (G.7) we then arrive at the expression in Equation (G.9). *Q.E.D.*

G.2. *Small Observation Radius*

Next, we consider the case of a small observation radius n_s. The probability that agent j receives a link from the entrant at time t, conditional on the sample $\mathcal{S}_t$ (and the current network G_{t-1}) when $\beta=0$ is given by

$$K_t^\beta(j|\mathcal{S}_t,G_{t-1})=\frac{m}{|\mathcal{S}_t|}\mathbb{1}_{\mathcal{S}_t}(j).$$

In the following, we assume that $\mathcal{S}_t\approx n_s(\bar{d}+1)$, where the average degree is given by $\bar{d}=2m$, so that $\mathcal{S}_t\approx n_s(2m+1)$. Note that this assumption is much stronger than the approximation we have made in Equation (3.4). The probability that an agent j receives a link from t is then given by

$$K_t^\beta(j|G_{t-1}) = \frac{m}{|\mathcal{S}_t|}\frac{n_s(1+d_{G_{t-1}}(j))}{t} + O\left(\frac{1}{t^2}\right) \approx \frac{m}{n_s(2m+1)}\frac{n_s(1+d_{G_{t-1}}(j))}{t} + O\left(\frac{1}{t^2}\right)$$

$$\approx \frac{m}{2m+1}\frac{1+d_{G_{t-1}}(j))}{t}. \tag{G.11}$$

An analysis following the recursive Equation (B.3) with the attachment kernel in Equation (G.11) yields the following proposition.

PROPOSITION 5 *Consider the sequence of degree distributions* $\{P_t\}_{t\in\mathbb{N}}$ *generated by an indefinite iteration of the network formation process* $(G_t^\beta)_{t\in\mathbb{N}}$ *of Definition 2 with* $\beta = 0$. *If* $n_s > 1$ *or* $m > 1$, *further assume that Equation (G.11) holds. Then, for all,* $k \geq 0$ *we have* $P_t(k) \to P(k)$, *where*

$$P(k) = \frac{(1+2m)\Gamma\left(3+\frac{1}{m}\right)}{m\Gamma\left(3+k+\frac{1}{m}\right)}. \tag{G.12}$$

PROOF OF PROPOSITION 5: Equation (G.12) follows directly from the recursion in Equation (B.3) and Equation (G.11). *Q.E.D.*

From Equation (G.12) we find that the degree distribution follows a power-law as $P(k) \sim k^{-\left(3+\frac{1}{m}\right)}$ for large k. For the dynamics of $k_s(t)$ in the continuum approximation we get with Equation (G.11) the following differential equation

$$\frac{dk_s(t)}{dt} = \frac{m}{2m+1}\frac{k_s(t)+1}{t}$$

with the solution

$$k_s(t) = (m+1)\left(\frac{t}{s}\right)^{\frac{m}{2m+1}} - 1 \tag{G.13}$$

The degree distribution in the continuum approximation is then given by[8]

$$P(k) = \frac{2m+1}{m}(m+1)^{2+\frac{1}{m}}(1+k)^{-\left(3+\frac{1}{m}\right)}, \tag{G.14}$$

satisfying the normalization condition $\int_0^\infty P(k)dk = 1$.

Next we consider the average nearest neighbor degree.

[8]Note that the approximation for the degree distribution in Equation (G.14) has also been obtained in Wang et al. (2009).

Proposition 6 *Consider the network formation process $(G_t^\beta)_{t\in\mathbb{R}_+}$ of Definition 2 in the continuum approximation with n_s small enough and assume that Equation (G.13) holds. If $\beta = 0$ then the nearest-neighbor degree distribution is given by*

$$\text{(G.15)}\quad k_{nn}(k) = \frac{1}{k}\left(\left(\frac{t}{s+1}\right)^a \left(a(m+1)^2 s^{2a-1} H_s^{2a} - 1\right) + (m+1)\left(\frac{t}{s}\right)^a \ln\left(\frac{t}{s+1}\right)^a\right),$$

where $a = \frac{m}{2m+1}$, $s = t\left(\frac{k+1}{m+1}\right)^{-\frac{1}{a}}$ and the Harmonic number is defined as $H_s^{2a} \equiv \sum_{j=1}^s \frac{1}{j^{2a}}$.

Proof of Proposition 6: Let $R_s(t) = \sum_{j\in\mathcal{N}_{G_t}(s)} k_j(t)$ be the sum of the degrees of the neighbors of vertex s at time t. Denoting by $a = \frac{m}{1+2m}$, we have up to leading orders in $O\left(\frac{1}{t}\right)$ that[9]

$$\begin{aligned}
\frac{dR_s(t)}{dt} &= \frac{n_s}{t}\sum_{j\in\mathcal{N}_{G_t}(s)} \frac{m}{|\mathcal{S}_t|} k_j(t) + \frac{n_s}{t}\sum_{j=1}^m j \frac{\binom{k_s(t)}{j}\binom{|\mathcal{S}_t|-k_s(t)}{m-j}}{\binom{|\mathcal{S}_t|}{m}} \\
&= \frac{a}{t}\left(k_s(t) + R_s(t)\right) = \frac{a}{t}\left((m+1)\left(\frac{t}{s}\right)^a - 1 + R_s(t)\right),
\end{aligned}$$

where we have assumed that $|\mathcal{S}_t| \approx n_s(2m+1)$ and used the relation $s = t\left(\frac{k+1}{m+1}\right)^{-\frac{1}{a}}$. The solution is given by

$$R_s(t) = 1 + \left(\frac{t}{s}\right)^a \left(R_s(s) - 1 + (m+1)\ln\left(\frac{t}{s}\right)^a\right),$$

and the initial condition is given by

$$R_{s+1}(s+1) = \frac{a}{s}\sum_{j=1}^s (1 + k_j(s))^2 = a(m+1)^2 s^{2a-1} H(s, 2a).$$

Using this equation to solve for C_s delivers Equation (G.15). *Q.E.D.*

Finally, we can compute the clustering coefficient as given in the following proposition.

Proposition 7 *Consider the network formation process $(G_t^\beta)_{t\in\mathbb{R}_+}$ of Definition 2 in the continuum approximation with n_s small enough and assume that Equation (G.13) holds. Let $a = \frac{m}{2m+1}$ and $b = \frac{2a(m-1)}{n_s(2m+1)-1}$ with $a > b > 0$. If $\beta = 0$ then the average clustering coefficient of an agent with degree k is bounded by $\underline{C}(k) \leq C(k) \leq \overline{C}(k)$, where*

$$\text{(G.16)}\quad \underline{C}(k) = \frac{2}{(a-b)k(k-1)}\left(a - (a+mb)\left(\frac{1+k}{1+m}\right)^{\frac{b}{a}} + bk\right),$$

[9] We ignore cases in which two or more neighbors of s are found as the neighbors of directly observed vertices (other than s), which happens with probability $O\left(\frac{1}{t^2}\right)$.

and

$$\overline{C}(k) = \frac{2}{(a-b)k(k-1)}\left(a + \left(\binom{m}{2}(a-b) - (a+mb)\right)\left(\frac{1+k}{1+m}\right)^{\frac{b}{a}} + bk\right), \tag{G.17}$$

and the property that $C(k) = O\left(\frac{1}{k}\right)$.

PROOF OF PROPOSITION 7: We need to consider the cases we have encountered already in the proof of Proposition 8 for a vertex s to form an additional triangle by an entrant t (see Figure B.3). The expected number of triangles associated with case (i) is given by

$$\frac{n_s}{t}\sum_{j=1}^{m-1} j\frac{\binom{k_s(t)}{j}\binom{|\mathcal{S}_t|-k_s(t)-1}{m-(j+1)}}{\binom{|\mathcal{S}_t|}{m}} = \frac{n_s}{t}\frac{m(m-1)k_s(t)}{(1+2m)n_s(n_s(1+2m)-1)},$$

where we have assumed that $|\mathcal{S}_t| = n_s(2m+1)$. Similarly, for case (ii) we get

$$k_s(t)\frac{n_s}{t}\frac{\binom{|\mathcal{S}_t|-2}{m-2}}{\binom{|\mathcal{S}_t|}{m}} = \frac{k_s(t)n_s}{t}\frac{m(m-1)}{|\mathcal{S}_t|(|\mathcal{S}_t|-1)} = \frac{k_s(t)}{t}\frac{m(m-1)}{(2m+1)(n_s(2m+1)-1)},$$

and for case (iii) we obtain

$$2M_s(t)\frac{n_s}{t}\frac{\binom{|\mathcal{S}_t|-2}{m-2}}{\binom{|\mathcal{S}_t|}{m}} = \frac{2M_s(t)n_s}{t}\frac{m(m-1)}{|\mathcal{S}_t|(|\mathcal{S}_t|-1)} = \frac{2M_s(t)}{t}\frac{m(m-1)}{(2m+1)(n_s(2m+1)-1)}.$$

Denoting by $a = \frac{m}{2m+1}$ and $b = \frac{2a(m-1)}{n_s(2m+1)-1}$ we can add cases (i), (ii) and (iii) to get

$$\frac{dM_s(t)}{dt} = \frac{2a(m-1)}{t(n_s(2m+1)-1)}(k_s(t)+M_s(t)) = \frac{b}{t}\left(\left((m+1)\left(\frac{t}{s}\right)^a - 1 + M_s(t)\right)\right).$$

Using as a lower bound for the initial condition $M_s(s) \geq 0$ and an upper bound $M_s(s) \leq \binom{m}{2}$ as well as $s = \left(\frac{1+k}{1+m}\right)^{-1/a} t$, we obtain the corresponding bounds for the clustering coefficient in Equations (G.16) and (G.17). Both bounds decay as $\frac{2b}{a-b}\frac{1}{k}$ for large k and their difference vanishes for large k, implying that also $C(k) = O\left(\frac{1}{k}\right)$. *Q.E.D.*

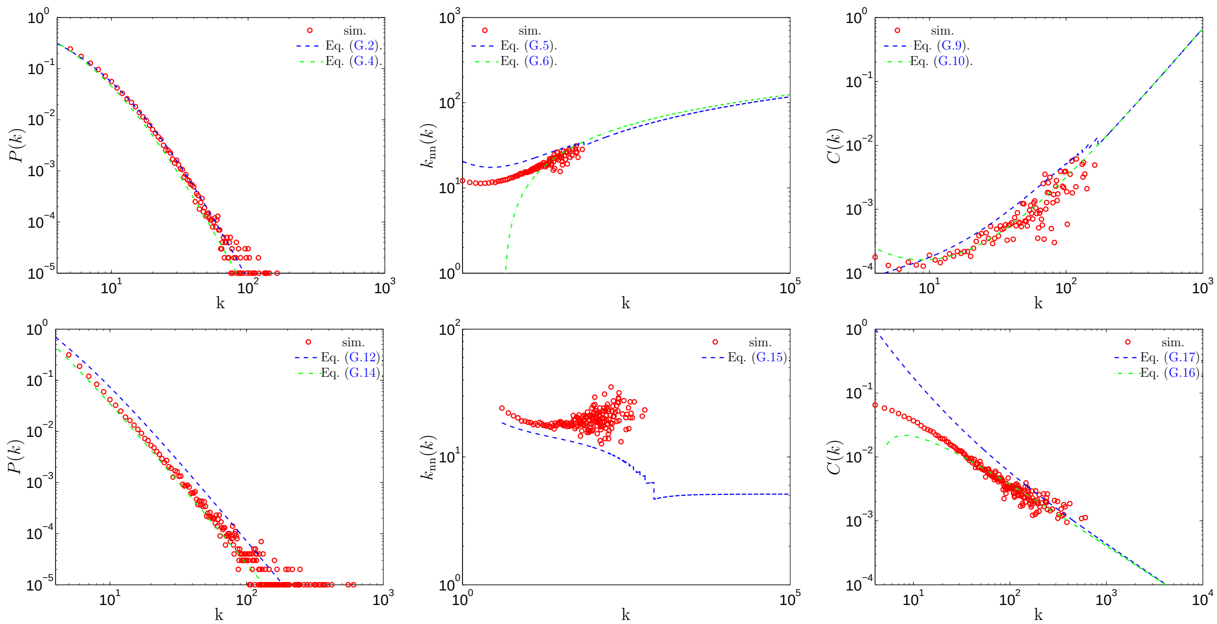

Figure G.1: (Top row) Comparison of simulation results with the theoretical predictions for $T = 10^5$, $\mathcal{S}_t = \mathcal{P}_{t-1}$ and $m = 4$ with $\beta = 0.1$ under the linear approximation to the attachment kernel. (Bottom row) Comparison of simulation results for $T = 10^5$ and $n_s = m = 4$ ($\beta = 0$) with the theoretical predictions. Comparing the results of global and local information, we find that they differ mainly in the clustering degree distribution.

H. HETEROGENEOUS LINKING OPPORTUNITIES

In this section we assume that not all agents become active during the network formation process. More precisely, we assume that only a fraction $p \in (0,1)$ of the population of agents forms links, while the remaining agents stay passive throughout the whole evolution of the network. We assume that initially, agents in $[T] = \{1, 2 \ldots, T\}$ are randomly assigned to sets $\mathcal{P}_1$ with probability $1-p$ and to $\mathcal{A}_1$ with probability p, such that $|\mathcal{A}_1| = \lfloor pT \rfloor$ and $|\mathcal{P}_1| = \lceil (1-p)T \rceil$. The agents in $[m]$ are all connected to each other and form a complete graph K_m. At time $t \leq m+1$ these agents are all in the set $\mathcal{P}_t$. The network evolution process is then defined as follows:

DEFINITION 3 *For a fixed $T \in \mathbb{N} \cup \{\infty\}$ we define a network formation process $(G_t)_{t \in [T]}$ as follows. Given the initial graph $G_1 = \ldots = G_{m+1} = K_{m+1}$, for all $t \in [T] \backslash \{1, \ldots, m+1\}$ the graph G_t is obtained from G_{t-1} by applying the following steps:*

Growth: *Given $\mathcal{P}_1$ and $\mathcal{A}_1$, for all $t > m$, if agent $t \in \mathcal{A}_{t-1}$ then the agent sets in period t are given by $\mathcal{P}_t = \mathcal{P}_{t-1} \cup \{t\}$ and $\mathcal{A}_t = \mathcal{A}_{t-1} \backslash \{t\}$, respectively. Otherwise, set $\mathcal{P}_t = \mathcal{P}_{t-1}$ and $\mathcal{A}_t = \mathcal{A}_{t-1}$.*

Network sampling: *If $t \in \mathcal{A}_{t-1}$ then t observes a sample $\mathcal{S}_t \subseteq \mathcal{P}_{t-1}$. The sample $\mathcal{S}_t$ is constructed by selecting $n_s \geq 1$ agents $i \in \mathcal{P}_{t-1}$ uniformly at random without replacement and adding i as well as the out-neighbors $\mathcal{N}^+_{G_{t-1}}(i)$ of i to $\mathcal{S}_t$.*

Link creation: *If $t \in \mathcal{A}_{t-1}$, given the sample $\mathcal{S}_t$, agent t creates $X_m \geq 1$, $\mathbb{E}(X_m) = m$ links to agents in $\mathcal{S}_t$ without replacement. For each link, agent t chooses the $j \in \mathcal{S}_t$ that maximizes $f^\delta_t(G_{t-1}, j) + \varepsilon_{tj}$.*

The number of links X_m to be created by an entrant is a discrete random variable with expectation $\mathbb{E}(X_m) = m$. The results and approximations we obtain in this section do not depend on the specific distribution we choose for X_m. We illustrate this by comparing our theoretical approximations with simulations for a uniform distribution $X_m \sim \mathrm{U}\{1, \ldots, 2m-1\}$ and a Poisson distribution $X_m \sim \mathrm{Pois}(m)$.

H.1. *Large Observation Radius*

We first consider the case of a large observation radius such that $\mathcal{S}_t = \mathcal{P}_{t-1}$ for all $t > m+1$. Similar to our discussion in Section 3.2, the probability that an agent $j \in \mathcal{P}_{t-1}$ with degree $d_{G_{t-1}}(j)$ receives a link by the entrant at time t up to leading orders in $O\left(\frac{1}{t}\right)$ is given by

$$K^\beta_t(j|G_{t-1}) \approx \frac{pm}{1+\beta pm} \frac{1+\beta d_{G_{t-1}}(j)}{t}. \tag{H.1}$$

Following the recursive Equation (B.3) with the attachment kernel in Equation (H.1) yields the following proposition.

PROPOSITION 8 *Consider the sequence of degree distributions $\{P_t\}_{t \in \mathbb{N}}$ generated by an indefinite iteration of the network formation process $(G^\beta_t)_{t \in \mathbb{N}}$ introduced in Definition 3 with*

n_s large enough such that $\mathcal{S}_t = \mathcal{P}_{t-1}$ for every $t > m+1$. Then, for all $k \geq m$ we have in the limit $\beta \to 0$ that $P_t^\beta(k) \to P^\beta(k)$ almost surely, where

$$P^\beta(k) = \frac{1+\beta mp}{1+mp(1+\beta)} \frac{\Gamma\left(\frac{1}{\beta}+k\right)\Gamma\left(2+\frac{1+mp}{\beta mp}\right)}{\Gamma\left(\frac{1}{\beta}\right)\Gamma\left(2+\frac{1+mp}{\beta mp}+k\right)}. \tag{H.2}$$

Proof of Proposition 8: Equation (H.2) follows directly from the recursion in Equation (B.3) and the attachment kernel in Equation (H.1). *Q.E.D.*

From the attachment kernel in Equation (H.1) we can write for the dynamics of the in-degree $k_s(t)$ of vertex s at time t in the continuum approximation

$$\frac{dk_s(t)}{dt} = \frac{pm}{1+\beta pm}\frac{1+\beta k_j(t)}{t},$$

with the initial condition $k_s(s) = 0$. The solution is given by

$$k_s(t) = \frac{1}{\beta}\left(\left(\frac{t}{s}\right)^{\frac{\beta pm}{1+\beta pm}} - 1\right), \tag{H.3}$$

and we obtain for the degree distribution in the continuum approximation

$$P^\beta(k) = \frac{1+\beta mp}{mp}(1+\beta k)^{-\left(2+\frac{1}{\beta mp}\right)}, \tag{H.4}$$

with $\int_0^\infty P^\beta(k)dk = 1$. For $p = 1$ we recover the distribution in Equation (B.13). The degree distribution from Equations (H.2) and (H.4) can be seen in Figure H.1.

Next we consider the average nearest neighbor degrees. We can state the following proposition.

Proposition 9 *Consider the network formation process $(G_t^\beta)_{t\in\mathbb{R}_+}$ of Definition 3 with $\mathcal{S}_t = \mathcal{P}_{t-1}$ for all $t > m+1$ in the continuum approximation and assume that Equation (H.3) holds. Then in the limit $\beta \to 0$ the nearest-neighbor degree distribution is given by*

$$k_{nn}^-(k) = \frac{1}{\beta^2 k}\left(1+(1+\beta k)(\ln(1+\beta k)-1)\right), \tag{H.5}$$

and the average nearest neighbor out-degree is given by

$$k_{nn}^+(k) = \frac{1}{\beta^2 m}\left(\left(\beta m(1+p(\beta-1)) + \frac{a}{s}s^{2a}\zeta(s,2a)\right)\left(\frac{t}{s+1}\right)^a - m\beta\right), \tag{H.6}$$

where $a = \frac{\beta mp}{1+\beta mp}$, $s = t(1+\beta k)^{-\frac{1}{a}}$.

Observe that Equation (H.5) is independent of p and identical to Equation (B.17) from Proposition 5. From Proposition 9 we find that for large k,f the average nearest in-neighbor connectivity grows logarithmically with k while the average nearest out-neighbor connectivity becomes independent of k and grows with the network sizes as $t^{\frac{\beta mp}{1+\beta mp}}$.

Proof of Proposition 9: Let $R_s^-(t) = \sum_{j\in\mathcal{N}^-_{G_t}(s)} k_j(t)$. Up to leading orders in $O\left(\frac{1}{t}\right)$ we then have that

$$\frac{dR_s^-(t)}{dt} = \sum_{j\in\mathcal{N}^-_{G_t}(s)} \frac{pm}{1+\beta pm}\frac{1+\beta k_j(t)}{t} = \frac{a}{t}\left(\frac{1}{\beta}k_j(t) + R_s^-(t)\right),$$

where we have denoted by $a = \frac{\beta mp}{1+\beta mp}$. The initial condition is given by $R_s^- = 0$. The solution is

$$R_s^-(t) = \frac{1}{\beta^2}\left(1 + \left(\frac{t}{s}\right)^a\left(a\ln\left(\frac{t}{s}\right) - 1\right)\right).$$

Using the fact that $\frac{t}{s} = (1+\beta k)^{\frac{1}{a}}$ from Equation (H.3), we obtain

$$R_s^-(t) = \frac{1}{\beta^2}\left(1 + (1+\beta k)(-1 + \ln(1+\beta k))\right).$$

With $k_{\mathrm{nn}}(k) = \frac{R_s^-}{k}$, the expression in Equation (H.5) follows.

Next we turn to the average nearest out-neighbor degree. Consider a vertex s which has received a linking opportunity upon entry. Let $R_s^+(t) = \sum_{j\in\mathcal{N}^+_{G_t}(s)} k_j(t)$. Then up to leading orders in $O\left(\frac{1}{t}\right)$ we obtain

$$\frac{dR_s^+(t)}{dt} = \sum_{j\in\mathcal{N}^+_{G_t}(s)} \frac{a}{t}\left(\frac{1}{\beta} + k_j(t)\right) = \frac{a}{t}\left(\frac{m}{\beta} + R_s^+(t)\right),$$

where $a = \frac{\beta pm}{1+\beta pm}$. The solution is given by

$$R_s^+(t) = -\frac{m}{\beta} + t^a C_s.$$

The constant C_s is determined by the initial condition

$$R_{s+1}^+ = \sum_{j=1}^{s}\frac{a}{s}\left(\frac{1}{\beta} + k_j(t)\right)(k_j(t)+1) = \frac{a}{\beta^2}\left(\beta - 1 + mp\beta(\beta-1) + s^{2a-1}H(s,2a)\right).$$

We then obtain

$$R_s^+(t) = \frac{1}{\beta^2}\left(\left(\beta m(1+p(\beta-1)) + \frac{a}{s}s^{2a}H(s,2a)\right)\left(\frac{t}{s+1}\right)^a - m\beta\right),$$

with $s = t(1+\beta k)^{-\frac{1}{a}}$ from Equation (H.3) and $k_{\text{nn}}^+ = \frac{R_s^+(k)}{m}$. *Q.E.D.*

Moreover, we can derive the clustering degree distribution.

Proposition 10 *Consider the network formation process $(G_t^\beta)_{t\in\mathbb{R}_+}$ of Definition 3 with $\mathcal{S}_t = \mathcal{P}_{t-1}$ for all $t > m+1$ in the continuum approximation and assume that Equation (H.3) holds. Then in the limit $\beta \to 0$ the clustering degree distribution is given by*

$$\begin{aligned} C(k) &= \frac{2}{(k+pm)(k+pm-1)}\frac{a(m-1)}{mp\beta^3 b^2 s}\left(sb^2\frac{mp\beta^3}{a(m-1)}M_s + \left((1+\beta k)^b - 1\right)\right. \\ &\quad \times \left.\left(b\left(\frac{s}{s+1}\right)^a\left(c + as^{2a-1}\zeta(s,2a)\right) - 1\right) + b(1+\beta k)^b \ln(1+\beta k)\right), \end{aligned} \tag{H.7}$$

where $a = \frac{\beta mp}{1+\beta mp}$, $b = 2 - \frac{1}{a}$, $c = \beta m(1+p(\beta-1))$, the initial condition is given by

$$M_{s+1} = \frac{mp(m-1)s^{2a-2}}{(1+\beta pm)^2}\left(\sum_{i=1}^m \frac{1}{i^a}\sum_{j=i+1}^m \frac{1}{j^a} + \frac{2mp}{1+\beta pm}\sum_{i=m+1}^s \frac{1}{i^{2a}}\sum_{j=i}^{s-1}\frac{1}{j}\right),$$

and $s = t(1+\beta k)^{-\frac{1}{a}}$.

For large k (and small s, respectively) the first term in the initial condition M_s dominates, and the behavior of the clustering coefficient is given by

$$C(k) \sim \frac{2t^{-2(1-a)}(1+k\beta)^{2\left(\frac{1}{a}-1\right)}}{(k+pm)(k+pm-1)}\frac{mp(m-1)}{(1+\beta pm)^2}\sum_{i=1}^m i^{-a}\sum_{j=i+1}^m j^{-a}. \tag{H.8}$$

We see that this expression grows with k as a power-law with exponent $2\left(\frac{1}{a}-2\right) = -2 + \frac{2}{mp\beta}$.[10] Moreover, we find that the clustering coefficient is decreasing with the network size as $t^{-2(1-a)} = t^{-\frac{2}{1+mp\beta}}$.

Proof of Proposition 10: We need to consider the same cases as in the proof of Proposition 7. The probability associated with case (i) in Figure B.2 is given by

$$\frac{pm(1+\beta k_s(t))}{(1+\beta pm)t}\sum_{j\in\mathcal{N}_{G_t}^+(s)}\frac{(m-1)(1+\beta k_j(t))}{(1+\beta pm)t} = \frac{pm(m-1)(1+\beta k_s(t))}{(1+\beta pm)^2 t^2}(m+\beta R_s^+).$$

[10] We need only consider values of k such that $C(k)$ does not exceed its upper bound given by one.

Similarly, for the probability of case (ii) in Figure B.2 we obtain

$$\frac{pm(1+\beta k_s(t))}{(1+\beta mp)t}\sum_{j\in\mathcal{N}^-_{G_t}(s)}\frac{(m-1)(1+\beta k_j(t))}{(1+\beta pm)t}=\frac{pm(m-1)(1+\beta k_s(t))}{(1+\beta pm)^2t^2}(k_s(t)+\beta R^-_s)$$

With R^+_s and R^-_s given by Equations (H.5) and (H.5), respectively, we obtain

$$\begin{aligned}\frac{dM_s(t)}{dt}&=\frac{pm(m-1)(1+\beta k_s(t))}{(1+\beta pm)t^2}(m+k_s(t)+\beta(R^+_s+R^-_s))\\&=\frac{a^2}{t^2}\frac{m-1}{pm\beta^3}\left(\left(c+as^{2a-1}H(s,2a)\right)\left(\frac{t}{s}\right)^a\left(\frac{t}{s+1}\right)^a+\left(\frac{t}{s}\right)^{2a}a\ln\left(\frac{t}{s}\right)^a\right),\end{aligned}$$

where we have denoted by $c=\beta m(1+p(\beta-1))$. The initial condition is given by

$$\begin{aligned}M_{s+1}&=p\frac{m(m-1)}{2}\sum_{j\neq i}^{s}\frac{1+\beta k_i(s)}{(1+\beta pm)s}\frac{1+\beta k_j(s)}{(1+\beta pm)s}\Big(\Theta(m+1-i)\Theta(m+1-j)\\&\quad+\Theta(i-j)\Theta(j-m)pm\frac{1+\beta k_j(i)}{(1+\beta pm)(i-1)}+\Theta(j-i)\Theta(i-m)pm\frac{1+\beta k_i(j)}{(1+\beta pm)(j-1)}\Big)\\&=\frac{mp(m-1)s^{2a-2}}{(1+\beta pm)^2}\left(\sum_{i=1}^{m}\frac{1}{i^a}\sum_{j=i+1}^{m}\frac{1}{j^a}+\frac{2mp}{1+\beta pm}\sum_{i=m+1}^{s}\frac{1}{i^{2a}}\sum_{j=i+1}\frac{1}{j-1}\right),\end{aligned}\tag{H.9}$$

where we have denoted by $a=\frac{\beta pm}{1+\beta pm}$. The initial condition M_{s+1} together with Equation (H.9) deliver

$$\begin{aligned}C(k)&=\frac{2}{(k+pm)(k+pm-1)}\frac{a(m-1)}{mp\beta^3b^2s}\left(sb^2\frac{mp\beta^3}{a(m-1)}M_s+\left((1+\beta k)^b-1\right)\right.\\&\quad\left.\times\left(b\left(\frac{s}{s+1}\right)^a\left(c+as^{2a-1}H(s,2a)\right)-1\right)+b(1+\beta k)^b\ln(1+\beta k)\right).\end{aligned}$$

Together with the initial condition, this is the expression in Proposition 10. *Q.E.D.*

Next, we turn to the analysis of the connectivity of the networks generated by our model. We consider only the simple case where $m=1$ and the limit of strong noise with $\beta\to 0$, where the network formation process follows a uniformly grown random graph.

PROPOSITION 11 *Let $N_s(t)$ denote the number of components of size s at time t. Consider the network formation process $(G^\beta_t)_{t\in\mathbb{N}}$ of Definition 3 with $\mathcal{S}_t=\mathcal{P}_{t-1}$ for all $t>m+1$. Assume that $m=1$ and $\beta=0$. If $p<1$, then there exists no giant component and the*

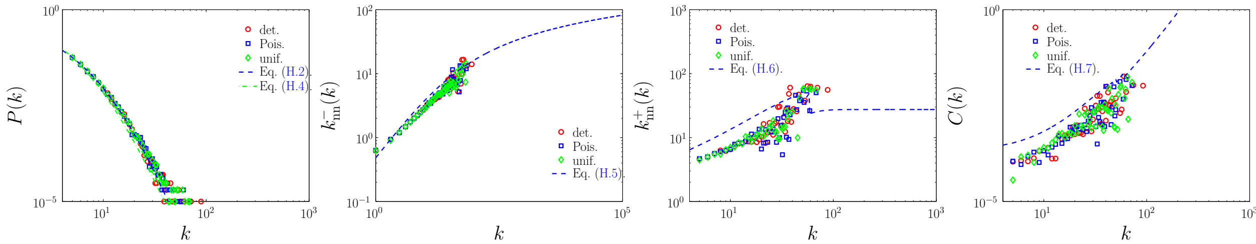

Figure H.1: Comparison of simulation results with theoretical prediction of the link formation process in Definition 3 under global information with $p = 0.5$, $m = 4$, $\beta = 0.1$ and $T = 10^5$. Simulation results for the deterministic case ($\circ$) a uniform distribution $X_m \sim \mathrm{U}\{1, 2m-1\}$ ($\diamond$) and a Poisson distribution $X_m \sim \mathrm{Pois}(m)$ ($\square$) both with expectation $\mathbb{E}(X_m) = m$ are shown.

asymptotic (finite) component size distribution $P(s) = \lim_{t\to\infty} \frac{N_s(t)}{t}$ *is given by*

$$P(s) = \frac{(1-p)\Gamma\left(\frac{1}{p}\right)\Gamma(s)}{p^2\Gamma\left(1+\frac{1}{p}+s\right)}. \tag{H.10}$$

When $p = 1$ *then there exists a giant component encompassing all nodes.*

Proof of Proposition 11: Let $N_s(t)$ denote the number of components of size s at time t. For $m = 1$, the entrant t forms only a single link and we need only consider the case of the component with size $s - 1$ to receive a link in the contribution to the growth of $N_s(t)$. It then follows that

$$\begin{aligned}
\mathbb{E}_t\left[N_1(t+1)|G_t\right] =& N_1(t) + (1-p) - p\frac{N_1(t)}{t}, \\
\mathbb{E}_t\left[N_s(t+1)|G_t\right] =& N_s(t) + p\frac{(s-1)N_{s-1}(t)}{t} - p\frac{sN_s(t)}{t}, \quad s \geq 2.
\end{aligned}$$

Denote by $n_s(t) = \frac{\mathbb{E}_t[N_s(t)]}{t}$. Taking expectations in the above equations delivers

$$\begin{aligned}
n_1(t+1)(t+1) =& n_1(t)t + (1-p) - pn_1(t), \\
n_s(t+1)(t+1) =& n_s(t)t + p(s-1)n_{s-1}(t) - psn_s(t), \quad s \geq 2.
\end{aligned}$$

For the stationary distribution $P(s) = \lim_{t\to\infty} n_s(t)$ we then get

$$\begin{aligned}
P(1) =& \frac{1-p}{1+p}, \\
P(s) =& \frac{p(s-1)}{1+ps}P(s-1), \quad s \geq 2.
\end{aligned}$$

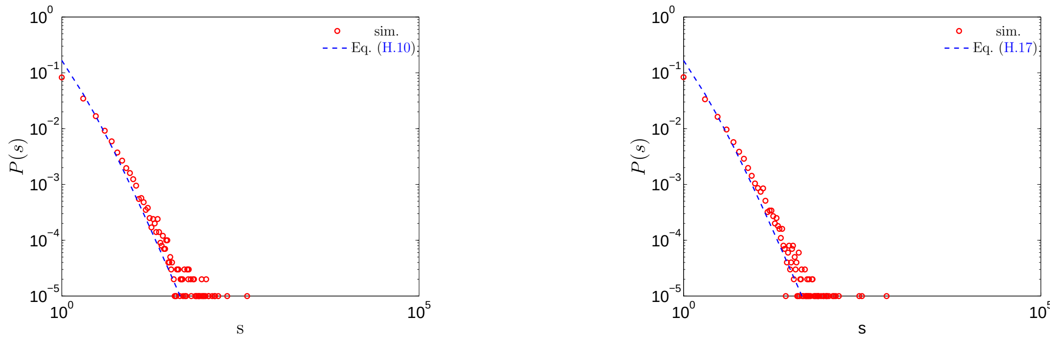

Figure H.2: Comparison of simulation results with theoretical predictions for the component size distribution $P(s)$ of the link formation process in Definition 3 under global information with $p = 0.5$, $m = 1$, $\beta = 0$ and $T = 10^5$ (left panel); with $p = 0.5$, $n_s = 1$, $m = 4$, $\beta = 0$ and $T = 10^5$ (right panel).

From this recursive equation we obtain

$$P(s) = P(1)p^{s-1}\prod_{k=2}^{s}\frac{k-1}{1+pk} = \frac{(1-p)\Gamma\left(\frac{1}{p}\right)\Gamma(s)}{p^2\Gamma\left(1+\frac{1}{p}+s\right)},$$

which is Equation (H.10).

We next consider the generating function of the component size distribution $g(x) = \sum_{s=1}^{\infty} sP(s)x^s$. Observe that $g(1) = \sum_{s=1}^{\infty} sP(s)$ the fraction of nodes in finite components. In the absence of a giant component (that grows with t), we must have that $g(1) = 1$. Inserting Equation (H.10) into $g(x)$ we find that $g(1) = 1$ as long as $p < 1$. Hence, the critical probability for the emergence of a giant component is $p = 1$. *Q.E.D.*

From Equation (H.10) we find that the component size decays as a power-law with exponent $1 + \frac{1}{p}$, i.e.

$$P(s) = \frac{1-p}{p^2}\Gamma\left(\frac{1}{p}\right)s^{-\left(1+\frac{1}{p}\right)}\left(1 + O\left(\frac{1}{s}\right)\right).$$

We finally note that when $\beta \to 0$, the probability that a component $H \in G_{t-1}$ of size s receives a link at time t, and thus grows by one, is given by

$$p\sum_{i\in H}\frac{1+\beta k_i(t)}{(1+\beta p)t} = \frac{p}{(1+\beta p)t}\sum_{i\in H}(s+\beta k_i(t)) \approx \frac{sp}{t},$$

where we have used the approximation $\sum_{i\in H} k_i(t) \approx sp$. This is the same probability for the growth of a component of size s as in the case of $\beta = 0$ and hence we obtain the same component size distribution as in Equation (H.10).

H.2. *Small Observation Radius*

Next, we consider the case of a small observation radius corresponding to small values of n_s. Similar to our discussion in Section 3.2, the probability that an agent $j \in \mathcal{P}_{t-1}$ with degree $d_{G_{t-1}}(j)$ receives a link by the entrant at time t up to leading orders in $O\left(\frac{1}{t}\right)$ is given by

$$K_t^{\beta}(j|G_{t-1}) \approx \frac{pm}{1+m}\frac{d_{G_{t-1}}(j)+1}{t}. \tag{H.11}$$

Using the recursive solution of Equation (B.3) we can state the following proposition.

PROPOSITION 12 *Consider the sequence of degree distributions* $\{P_t\}_{t\in\mathbb{N}}$ *generated by an indefinite iteration of the network formation process* $(G_t^{\beta})_{t\in\mathbb{N}}$ *of Definition 3 with* $\beta = 0$*. Further assume that Equation (H.11) holds. Then, for all,* $k \geq 0$ *we have* $P_t(k) \to P(k)$*, where*

$$P(k) = \frac{(1+m)k!\Gamma\left(2+\frac{m+1}{mp}\right)}{(1+m(1+p))\Gamma\left(2+\frac{m+1}{mp}+k\right)}. \tag{H.12}$$

PROOF OF PROPOSITION 12: Equation (H.12) follows directly from the recursion in Equation (B.3) and Equation (H.11). *Q.E.D.*

With Equation (H.11) it follows for the dynamics of $k_s(t)$ in the continuum approximation

$$\frac{dk_s(t)}{dt} = \frac{pm}{m+1}\frac{k_s(t)+1}{t},$$

with the solution

$$k_s(t) = \left(\frac{t}{s}\right)^{\frac{pm}{1+m}} - 1. \tag{H.13}$$

The degree distribution in the continuum approximation is then given by

$$P(k) = \frac{1+m}{pm}(1+k)^{-\left(1+\frac{1+m}{pm}\right)}, \tag{H.14}$$

with $\int_0^\infty P(k)dk = 1$. For large k, Equations (H.12) and (H.14) are equivalent. Moreover, for $p = 1$ we recover the distribution in Equation (B.15). Next we turn to the analysis of the average nearest neighbor degree.

PROPOSITION 13 *Consider the network formation process* $(G_t^{\beta})_{t\in\mathbb{R}_+}$ *of Definition 3 in the continuum approximation with* n_s *small enough and assume that Equation (H.13) holds. If*

$\beta = 0$ then the average nearest in-neighbor degree distribution is given by

$$k^-_{nn}(k) = \frac{1}{k}\left(1 + (k+1)(\ln(k+1) - 1)\right) \tag{H.15}$$

and the average nearest out-neighbor degree distribution is given by

$$k^+_{nn}(k) = \frac{mp+1}{m+1}k + \frac{p}{m+1}t^{2a-1}(k+1)^{-\frac{2a-1}{a}}\zeta(t(k+1)^{-\frac{1}{a}}, 2a) \tag{H.16}$$

where $a = \frac{mp}{1+m}$.

Proof of Proposition 13: In order to derive Equation (H.15), let us denote by $R^-_s(t)$ the sum of the in-neighbors' degrees of a vertex s at time t. We then have that

$$\frac{dR^-_s(t)}{dt} = \sum_{j \in \mathcal{N}^-_{G_t}(s)} \frac{a}{t}(1 + k_j(t)) = \frac{a}{t}\left(\left(\frac{s}{t}\right)^a - 1 + R^-_s(t)\right),$$

where we have denoted by $a = \frac{mp}{1+m}$. The initial condition is $R^-_s(s) = 0$. The solution is given by

$$R^-_s(t) = 1 + (k+1)(\ln(k+1) - 1),$$

where we have used the fact that $s = t(k+1)^{-\frac{1}{a}}$ from Equation (H.13). Noting that $k^-_{\mathrm{nn}}(k) = \frac{R^-_s}{k}$ we readily obtain Equation (H.15).

Next, we consider the out-neighbors of s. Assume that vertex s has out-degree m and denote by R^+_s the sum of the in-degrees of the out-neighbors of s at time t. We then can write

$$\frac{dR^+_s(t)}{dt} = \sum_{j \in \mathcal{N}^+_{G_t}(s)} \frac{a}{t}k_j(t) + p\frac{n_s}{t}\sum_{k=1}^{m} k\frac{\binom{m}{k}\binom{n_s(m+1)}{m-k}}{\binom{n_s(m+1)}{m}} = \frac{a}{t}\left(R^+_s(t) + \frac{m(mp+1)}{m+1}\right),$$

The solutions is given by $R^+_s(t) = -\frac{m(1+mp)}{1+m} + C_s t^a$ and the initial condition is

$$R^+_s(s) = \sum_{j=1}^{s} \frac{a}{s}(1 + k_j(s))^2 = as^{2a-1}H(s, 2a),$$

so that we get

$$R^+_s(t) = \frac{m(mp+1)}{m+1}\left(\left(\frac{t}{s}\right)^a - 1\right) + as^{2a-1}H(s, 2a).$$

Inserting $s = t(k+1)^{-\frac{1}{a}}$ from Equation (H.13) and using the fact that $k_{\mathrm{nn}}(k) = \frac{R^+_s}{m}$ delivers Equation (H.16) . *Q.E.D.*

In a similar fashion as in Proposition 8 we can also compute the clustering degree distribution.

Proposition 14 *Consider the network formation process* $(G_t^{\beta})_{t\in\mathbb{R}_+}$ *of Definition 3 in the continuum approximation with* n_s *small enough and assume that Equation (H.13) holds. If* $\beta = 0$ *then the average clustering coefficient of an agent with degree* k *is given by Proposition 8 setting* $a = \frac{mp}{m+1}$.

Proof of Proposition 14: We need to consider the same cases as in the proof of Proposition 8. We take $|\mathcal{S}_t| = n_s(m+1)$ ignoring terms of the order $O\left(\frac{1}{t^2}\right)$. For the probability of case (i) we obtain

$$p\frac{n_s}{t}\sum_{k=1}^{m-1} k\frac{\binom{m}{k}\binom{((n_s-1)(m+1))}{m-(k+1)}}{\binom{n_s(m+1)}{m}} = p\frac{m^2(m-1)}{(m+1)(n_s(m+1)-1)t}.$$

For case (ii) we get

$$pk_s(t)\frac{n_s}{t}\frac{\binom{n_s(m+1)-2}{m-2}}{\binom{n_s(m+1)}{m}} = p\frac{k_s(t)}{t}\frac{m(m-1)}{n_s(m+1)(n_s(m+1)-1)}.$$

and similarly, for case (iii) we get

$$pM_s(t)\frac{n_s}{t}\frac{\binom{n_s(m+1)-2}{m-2}}{\binom{n_s(m+1)}{m}} = p\frac{M_s(t)}{t}\frac{m(m-1)}{(m+1)(n_s(m+1)-1)}.$$

The dynamics of $M_s(t)$ is then given by

$$\begin{aligned}\frac{dM_s(t)}{dt} &= \frac{a(m-1)}{t(n_s(m+1)-1)}(m+k_s(t)+M_s(t)) \\ &= \frac{b}{t}(m+k_s(t)+M_s(t)) = \frac{b}{t}(m+\left(\frac{t}{s}\right)^a - 1 + M_s(t)),\end{aligned}$$

with $a = \frac{mp}{m+1}$. This differential equation is identical to (B.27) and hence we obtain the same result as in Proposition 8. *Q.E.D.*

In the following we study the connectivity of the emerging networks in the network formation process introduced in Definition 3. We restrict our analysis to the case of $n_s = 1$. Observe that the probability that a component of size s grows by one unit due to the attachment of an entrant t is equivalent to the event that t observes one of the nodes in the component when constructing the sample $\mathcal{S}_t$. The probability of this event is $\frac{ps}{t}$. Hence, we obtain the same component size distribution as in Proposition 11. We then can state the following proposition.

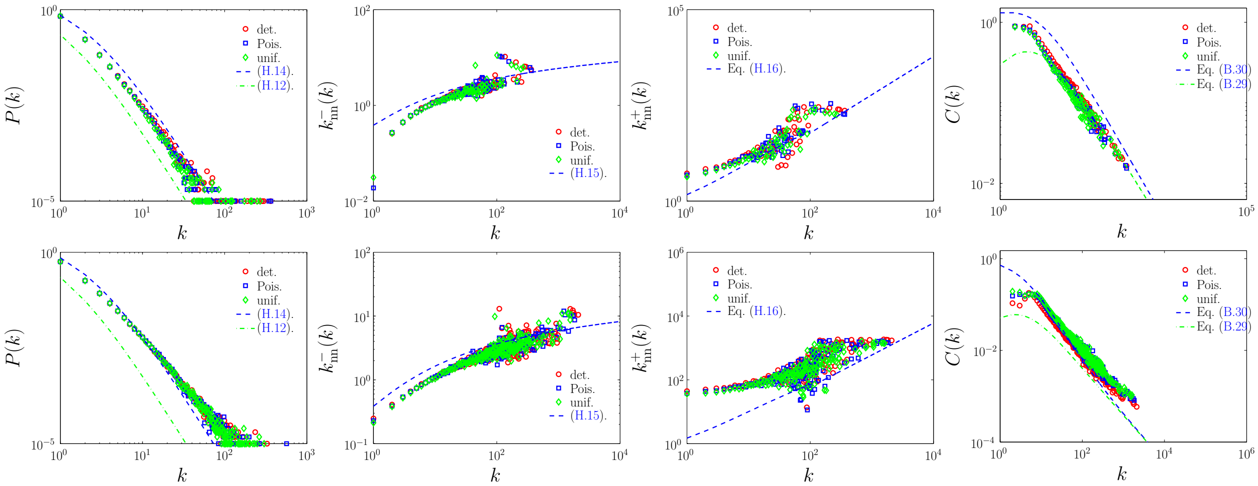

Figure H.3: Comparison of simulation results with theoretical predictions of the link formation process in Definition 3 with $p = 0.5$, $n_s = 1$, $m = 4$, $\beta = 0$ where the network size is $T = 10^5$ (top row) or $T = 2 \times 10^5$ (bottom row). We show simulations for the deterministic case ($\circ$), a uniform distribution $X_m \sim \mathrm{U}\{1, 2m-1\}$ ($\diamond$) and a Poisson distribution $X_m \sim \mathrm{Pois}(m)$ ($\square$) both with expectation $\mathbb{E}[X_m] = m$.

PROPOSITION 15 *Let $N_s(t)$ denote the expected number of components of size s at time t. Consider the network formation process $(G_t^{\beta})_{t \in \mathbb{N}}$ of Definition 3 with $n_s = 1$. Then the asymptotic component size distribution $P(s) = \lim_{t\to\infty} \frac{N_s(t)}{t}$ is given by*

$$P(s) = \frac{(1-p)\Gamma\left(\frac{1}{p}\right)\Gamma(s)}{p^2\Gamma\left(1 + \frac{1}{p} + s\right)}. \tag{H.17}$$

PROOF OF PROPOSITION 15: The proof follows the one of Proposition 11. *Q.E.D.*

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