Supplementary material for:
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Theoretical Economics 6(3)

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1 Proof of Proposition 1

Assume (1). That each $\succcurlyeq_{t,\omega}$ is complete and transitive is immediate; also, it is enough to verify that Axiom 4.1 (DC) holds for $a, b \in A_t(\omega)$ that agree everywhere except at a node $(t+1, \omega^+)$ that follows (t, ω): that is, $a(\omega') = b(\omega')$ for all $\omega' \in \mathscr{F}_t(\omega) \setminus \mathscr{F}_{t+1}(\omega^+)$, with $\omega^+ \in \mathscr{F}_t(\omega)$. Thus, suppose that $a(\omega^+) \succcurlyeq_{t+1,\omega^+} b(\omega^+)$; an easy induction argument then implies that DC holds for general $a, b \in A_t(\omega)$. By Bayesian updating, there is $p \in F_0^p$ such that $a(\omega^+)_{t+1,\omega^+} p \succcurlyeq b(\omega^+)_{t+1,\omega^+} p$; by Axiom 5.2 (Postulate P2), $\{a\}_{t,\omega} p = a(\omega^+)_{t+1,\omega^+} \big(\{a\}_{t,\omega} p\big) \succcurlyeq b(\omega^+)_{t+1,\omega^+} \big(\{a\}_{t,\omega} p\big) = \{b\}_{t,\omega} p$, where the second equality follows from the assumption that $a(\omega') = b(\omega')$ for $\omega' \in \mathscr{F}_t(\omega) \setminus \mathscr{F}_{t+1}(\omega^+)$. But then, by Bayesian updating, $\{a\} \succcurlyeq_{t,\omega} \{b\}$. Now the assertion of Postulate P2 also holds with $\succcurlyeq$ replaced by $\succ$ (if not, exchange the role of q and p to obtain a contradiction). Hence, the argument just given also shows that $a(\omega^+) \succ_{t+1,\omega^+} b(\omega^+)$ implies $\{a\} \succ_{t,\omega} \{b\}$. Hence (2) holds.

Conversely, assume that (2) holds. I first show that Axiom 4.2 (Postulate P2) and Bayesian updating hold for $t = 1$. If $r, s \in F_1^p(\omega)$ and $p = \{c\} \in F_0^p$, with $c \in A_1$, then there are $a, b \in A_0$ such that $a(\omega') = r$ and $b(\omega') = s$ for $\omega' \in \mathscr{F}_1(\omega)$, and $a(\omega') = b(\omega') = c(\omega')$ for $\omega' \in \Omega \setminus \mathscr{F}_1(\omega)$.

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Thus, if $r \succcurlyeq_{1,\omega} s$, DC implies that $r_{1,\omega}p \succcurlyeq s_{1,\omega}p$; conversely, if $r_{1,\omega}p \succcurlyeq s_{1,\omega}p$, then DC implies that $r \prec_{1,\omega} s$ would lead to a contradiction. Hence, Bayesian updating holds. Furthermore, suppose $r_{1,\omega}p \succcurlyeq s_{1,\omega}p$, so $r \succcurlyeq_{1,\omega} s$; consider $p' = \{c'\} \in F_0^p$ and $a', b' \in A_0$ such that $a'(\omega') = r$, $b'(\omega') = s$ if $\omega' \in \mathscr{F}_1(\omega)$ and $a'(\omega') = b'(\omega') = c'(\omega')$ otherwise. Then DC implies that $r_{1,\omega}q \succcurlyeq s_{1,\omega}q$, i.e. P2 also holds.

Now note that the above argument goes through if $\succcurlyeq$ and $\succcurlyeq_{1,\omega}$ are replaced with $\succcurlyeq_{t,\omega}$ and $\succcurlyeq_{t+1,\omega^+}$ respectively, where $\omega^+ \in \mathscr{F}_t(\omega)$: hence, DC implies that "one-step-ahead" versions of P2 and Bayesian updating hold for every $t = 0, \dots, T-2$. Now consider an arbitrary node (t,ω), and plans $r, s \in F_t^p(\omega)$ and $p, q \in F_0^p$. First, denote by $[t,\omega,(a_0,\dots,a_{t-1})]$ and $[t,\omega,(b_0,\dots,b_{t-1})]$ the unique histories in $r_{t,\omega}p$ and $s_{t,\omega}p$ corresponding to (t,ω). Note that $a_{t-1}(\omega') = r$, $b_{t-1}(\omega') = s$ for $\omega' \in \mathscr{F}_t(\omega)$, and $a_{t-1}(\omega') = b_{t-1}(\omega')$ for $\omega' \in \mathscr{F}_{t-1}(\omega) \setminus \mathscr{F}_t(\omega)$. The claim just established implies that $r \succcurlyeq_{t,\omega} s$ iff $\{a_{t-1}\} \succcurlyeq_{t-1,\omega} \{b_{t-1}\}$, and by induction $r \succcurlyeq_{t,\omega} s$ iff $r_{t,\omega}p = \{a_0\} \succcurlyeq \{b_0\} = s_{t,\omega}p$. In turn, this implies that Bayesian updating and P2 hold. ■