Supplement to Credible Ratings

Ettore Damiano, Li, Hao

University of Toronto

Wing Suen

The University of Hong Kong

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In this supplement we give two examples that illustrate how changes in the correlation between clients' qualities may have ambiguous effect on the equilibrium threshold in an inflationary equilibrium.

Example 1. Let $N = 2$, $\pi_0 = \pi_2 = \frac{1}{2}(1-\alpha)$, and $\pi_1 = \alpha$ for some $\alpha \in (0,1)$. Note that the correlation among the two clients qualities decreases in α while the ex-ante expected quality of each client is constant and equal to $\frac{1}{2}$. For each small ϵ, there is some $z \in (0, 1-\epsilon)$ such that

$$\frac{\alpha}{\alpha + \frac{1}{2}(1-\alpha)z} = \frac{\frac{1}{2}(1-\alpha)}{\frac{1}{2}(1-\alpha) + \frac{1}{2}(1-\alpha)(1-\epsilon-z)}.$$

Then with utility functions $U(B, g, \cdot)$ and $U(B, b, \cdot)$ such that q^* equals the value of the above expression, the reporting strategy in state 0 given by $p(0;0) = \epsilon$, $p(1;0) = z$ and $p(2;0) = 1-\epsilon-z$, together with reporting truthfully when at least one client has good quality, constitutes an inflationary equilibrium with full support.

Next, consider an increase in the value of α. In any full support inflationary equilibrium, the market belief after observing two good ratings is given by

$$\frac{\pi_2}{\pi_2 + \pi_0 p(2;0)} = \frac{\frac{1}{2}(1-\alpha)}{\frac{1}{2}(1-\alpha) + \frac{1}{2}(1-\alpha)p(2;0)}$$

and does not depend on the value of α. Thus, in any full support inflationary equilibrium, we must have $p(2;0) = 1-\epsilon-z$. On the contrary, the market belief after observing one good rating is strictly increasing in α and strictly decreasing in the value of $p(1;0)$. Thus, the value of $p(1;0)$ that makes the market belief after observing one good rating equal to q^* grows with α, and when it exceeds $z+\epsilon$ the equilibrium threshold must be positive.

Example 2. Let $N = 3$, $\pi_0 = \pi_3 = \frac{1}{2}(1-\alpha)$, and $\pi_1 = \pi_2 = \frac{1}{2}\alpha$. Note that the correlation among clients' qualities decreases in α while the ex-ante expected quality of each client is constant and equal to $\frac{1}{2}$. To construct an inflationary equilibrium with $l = 2$, consider a reporting strategy such that the agency issues both three good ratings and two good ratings with positive probability when there is one good quality client (i.e. $0 < p(2;1) < 1$ and $p(3;1) = 1 - p(2;1)$). The characterization in Lemma 3 then implies that $p(3;3) = p(2;2) = p(3;0) = 1$, and the market belief upon observing three and two good ratings are given by

$$q(3) = \frac{\frac{1}{2}(1-\alpha) + \frac{1}{2}\alpha p(3;1)\frac{1}{3}}{(1-\alpha) + \frac{1}{2}\alpha p(3;1)}$$

and

$$q(2) = \frac{\frac{1}{2}\alpha + \frac{1}{2}\alpha p(2;1)\frac{1}{2}}{\frac{1}{2}\alpha + \frac{1}{2}\alpha p(2;1)}$$

respectively. Note that $q(3) < q(2) < 1$ for any positive value of $p(2;1)$ and $p(3;1)$. Thus, for any utility function that satisfies

$$2U(B,b,0)+U(G,g,1) \leq U(B,b,0)+U(B,g,q(2))+U(G,g,q(2)) = 2U(B,g,q(3))+U(G,g,q(3)),$$

the reporting strategy described above constitutes an inflationary equilibrium. Let specify the utility function so that the first inequality above holds with the equal sign and suppose α increases. Note that the market belief after observing two good ratings is unaffected while the market belief after observing three good ratings decreases. While the reporting strategy described before is no longer an equilibrium, there is an equilibrium such that with one good quality client the agency issues two good ratings with the same probability as before, issues three good ratings with probability slightly lower than before and issues one good rating with the residual probability. Thus the equilibrium threshold decreases after an increase in α.