

## Complementarities in information acquisition with short-term trades

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In a financial market where agents trade for short-term profit and where news can increase the uncertainty of the public belief, there are strategic complementarities in the acquisition of private information and, if the cost of information is sufficiently small, a continuum of equilibrium strategies. Imperfect observation of past prices reduces the continuum of Nash equilibria to a Strongly Rational-Expectations Equilibrium. In that equilibrium, there are two sharply different regimes for the evolution of the price, the volume of trade, and information acquisition.

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**JEL CLASSIFICATION.** D53, D82, D83, G14.

### 1. INTRODUCTION

In trading financial assets, private information is valued because it enables traders to “beat the market.” When more agents acquire information, this value is reduced by the diffusion of the market. A formalization of this straightforward property is provided by Grossman and Stiglitz (1980) in a model where (i) agents hold their position until the revelation of the fundamental value of the asset and (ii) the structure of information is Gaussian.

Here, private information may be more valuable when more agents are informed. The mechanism rests only on the informative properties of the market, with no structural payoff externality. Key assumptions are that agents hold their positions only for the short-term and the structure of information is not Gaussian because some news can increase uncertainty.

When agents trade for the short-run, what matters is not the fundamental but the price of the asset in the near future. For private information to yield significant profit, the price has to move in the near term. When the price is driven by the trades of informed agents, a higher mass of these agents generates wider price movements, which may enhance the value of private information.

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Two effects take place. First, more informed agents today drive the price closer to the fundamental and reduce the value of private information. This is the effect of Grossman–Stiglitz. Second, a price movement today may reduce the confidence of the market and generate wider fluctuations in the future, say next period, for a given level of private information in that period. Why? In any period, the price is driven by the trade of newly informed agents (which is obviously a garbled signal), weighted against the public belief from the history of the market. When the market is more “uncertain,” that weight from history is reduced with respect to the new trades, and the price moves more. This effect cannot take place in a Gaussian framework where *any* new information reduces the variance of public information and increases the weight of history. When this second effect dominates the first one, a higher level of private information increases the value of private information and there is strategic complementarity in the acquisition of private information. The strategic complementarity may produce multiple equilibria, switches between sharply differentiated regimes of price fluctuations and trading volume, and “trade frenzies.”

To prove the argument, we need a framework where some news can increase the uncertainty of the information. This property arises in a setting where the fundamental is either in a “normal” state or in a state where a “shock” has occurred in which the value of the fundamental is sharply different.<sup>1</sup> In this case, it may be taken from an unbounded distribution, but for simplicity and without any loss of generality, this distribution is reduced to one point. By assumption, the fundamental takes one of two values,  $\theta_1$  and  $\theta_0$ , which are normalized to 1 and 0. The confidence of the market is therefore measured by the variance of the belief (i.e. the probability of state  $\theta_1 = 1$ ), which at the price  $p$  is  $p(1 - p)$ . The interesting case arises when the confidence is high, that is when the price is near 1 (or 0) where bad (good) news reduces (increases) the price and augments the variance.

Agents must use rationally the trade information in a properly functioning financial market.<sup>2</sup> The simplest model with this property is perhaps the model of **Glosten and Milgrom (1985)**. Two features are added: (i) agents who have private information hold the asset only for one period (but the model could be extended to holdings for a few periods); (ii) some agents, called information agents, can obtain information about the fundamental, i.e. invest, at some cost, before entering the market, and their decision depends on the information<sup>3</sup> publicly available at that time.

<sup>1</sup>For previous studies that depart from the Gaussian framework to generate time-variable uncertainty see, among others, **Detemple (1991)**, **David (1997)**, **Veronesi (1999)**.

<sup>2</sup>In **Froot et al. (1992)**, trade orders are executed randomly in the present and in the next period because of some ad hoc friction. Information about the fundamental is useful in predicting the information of others who boost the demand and the price in the next period when, by assumption, the other half of the orders is executed. Agents can learn (at no cost) only one of the two independent components of the fundamental. There is strategic complementarity on the choice of the signal.

<sup>3</sup>**Dow and Gorton (1994)** analyze the efficiency of financial markets with short-term trading and exogenous information. In a model of the Glosten–Milgrom type, they make the key assumption that the probability of an informed agent increases exogenously as the maturity of the asset goes to zero. There is a fixed cost of trading. When the maturity is long, the probability that the price moves in the right direction in the next period (because of the occurrence of an informed trader) is small and because of the fixed cost,

The model of sequential trade enables us to derive analytical results. In order to show the robustness of these results, a model is presented in the [Appendix](#) with a continuum of agents, informed and not informed, who simultaneously place limit orders in each period. A numerical analysis shows that the main property of strategic complementarity holds.

Two periods are the minimum with short-term trade and we begin with such a model in [Section 2](#) in order to show that strategy complementarity may arise between the levels of investment in the same period. If the agent trading in the first period happens to be an information agent, he makes a decision whether to get information about the fundamental before entering the market. His strategy is the probability  $\lambda$  to invest in information. Since  $\lambda$  depends on the public probability of  $\theta = 1$ , it is known by the market maker, who adjusts the ask and bid accordingly as in the standard Glosten–Milgrom model with asymmetric information and perfect competition. We analyze how the payoff of investment for an information agent depends on  $\lambda$  and the public belief about the fundamental. We show that if the “consensus is strong” before period 1, i.e., if the public probability of  $\theta = 1$  is near 1 or 0, an increase in  $\lambda$  increases the payoff of information.

There is an evident strategic complementarity from the future back to the present: an increase of information investment in period 2 (or  $t + 1$  in a general model), has a positive impact on the magnitude of the variation of the price in that period and therefore a positive impact on the value of information in the previous period.

In order to take into account the interactions between the levels of investment in different periods, the model is extended in [Section 3](#) to an infinite number of periods where the fundamental is revealed in any period with a vanishingly small probability.

In [Section 4](#), because of the richness of the set of equilibrium strategies, we consider time-invariant strategies that depend on the last transaction price. As in the two-period model, if the public belief is sufficiently near one or zero, the simultaneous investments in private information by different agents exhibit strategic complementarity. If the cost of information is sufficiently small, [Proposition 5](#) shows that there is a continuum of equilibrium strategies where agents follow a trigger strategy: they invest in period  $t$  if and only if the last observed price  $p_{t-1}$  is in some interval  $(p^{**}, p^*)$ ; the values  $p^{**}$  and  $p^*$  that define the trigger strategy are arbitrary within some intervals.

The continuum of equilibria under common knowledge opens the issue of robustness to a perturbation, and the problem of “equilibrium selection.” The model is therefore extended in [Section 5](#) with the very plausible assumption that agents, before they decide whether to get information about the fundamental, observe the last transaction price with small noise. The model is similar to a “global game” ([Carlsson and Damme 1993](#)), with two differences: market-makers have perfect information about the last transaction price, as suits their specialization, and more important, the iterated elimination of dominated strategies cannot be applied period-by-period separately as in

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agents do not trade. Trade begins only when the maturity is sufficiently short. [Vives \(1995\)](#) analyzes the informational content of prices with short-term traders in the CARA–Gauss model when private information is accrued over time and when the fundamental is revealed at the end of the  $N$ -period game.

standard models. Since the payoff of investment in any period  $t$  depends on the strategy of other agents in period  $t + 1$ , the iteration has to be implemented backwards and simultaneously for an arbitrarily large number of periods.

Under a vanishingly small observation noise, there is a unique trigger strategy that survives the iterated elimination of dominated strategies and is therefore a Strongly Rational-Expectations Equilibrium (SREE). The relevance of trigger equilibrium strategies is thus validated. They show that the existence of a continuum of equilibria depends on common knowledge and is not robust to a perturbation. However, the essential property in multiple equilibria is a discontinuity in the behavior of agents, and that property is strongly validated in Section 5 where, in the unique equilibrium, the information investment varies between zero and its maximum as the price crosses an interval that can be arbitrarily small. With vanishing noise, the level of investment jumps when the price crosses a threshold value, and the average amplitude of price changes between periods changes abruptly.<sup>4</sup>

## 2. A TWO-PERIOD MODEL

There is a financial asset a unit of which is a claim on the *fundamental*  $\theta$ , which is set by nature before the first period and equal to  $\theta_1$  with probability  $\mu$ , and to  $\theta_0$  with probability  $1 - \mu$ . Without loss of generality, these values are taken to be  $\theta_1 = 1$  for the “good” state and  $\theta_0 = 0$  for the “bad” state. As explained in the *Introduction*, the model could admit an unbounded distribution of values for the fundamental. The state is invariant over time. It is not directly observable and is revealed only after the second period.

The financial asset is traded in a setting that builds on the model of *Glosten and Milgrom (1985)*, and which is extended along that line with an infinite number of periods in the next section. In the first period, a new agent meets a risk-neutral profit-maximizing market-maker, and either trades one unit of the asset or does not trade. The new agent is of one of the following three types.

- (i) With probability  $\alpha \geq 0$ , the agent has exogenous private information about the true state. To simplify, and without loss of generality,<sup>5</sup> such an agent is perfectly informed about  $\theta$ . In some specific cases,  $\alpha$  is strictly positive.
- (ii) With probability  $\bar{\beta}$ , the agent is an *information agent* who can get, at a fixed cost  $c$ , information about the true state  $\theta$  before trading. As for the agents of the previous type, this information is assumed to be perfect: if the agent pays the cost  $c$ , he is said to *invest* and he gets to know  $\theta$ . The investment decision is made at

<sup>4</sup>In *Veronesi (1999)*, agents are risk-averse and there is a non-linear relation between the asset price and the public belief about the fundamental. Volatility depends on the asset price. Because of the risk-aversion, bad news when agents are fairly confident about a high fundamental has a strong impact because it reduces this confidence; good news has a weak impact because it also increases the uncertainty. In the present model, agents are risk-neutral and the price is always equal to the expected value of the fundamental.

<sup>5</sup>Imperfectly informed traders may be crowded out of the market into the bid-ask spread, and the analysis of the equilibrium may be more technical, without additional insight.

the beginning of the first period, knowing the public belief  $\mu$  about  $\theta$ . The strategy of an agent is defined by the probability to invest,  $\beta_1/\bar{\beta}$  with  $\beta_1 \in [0, \bar{\beta}]$ , and the parameter  $\beta_1$  denotes the strategy. An informed agent trades according to his perfect information, at any price: he buys (sells) when the state is good (bad). An information agent who is not informed does not trade, because of the spread between the ask and the bid, in equilibrium. In this section, we focus on the value of information and assume that  $\beta_1$  is fixed.

- (iii) With probability  $1 - \alpha - \bar{\beta}$ , the agent trades for an exogenous liquidity motive at any price. He sells, buys one unit, or does not trade, each with probability  $\frac{1}{3}$ . The issue of endogenous liquidity or hedging traders is discussed briefly in [Section 4](#).

An information agent in the first period trades for the short-term: he holds his position for only one period. His payoff is  $(E[p_2] - p_1)x$ , where  $x = 1$  if he buys,  $x = -1$  if he sells,  $p_1$  and  $p_2$  are the prices of the asset in the two periods, and the expectation  $E[p_2]$  is conditional on the information of the agent. All trades take place with a market-maker who is perfectly competitive with other market-makers, holds his position until the revelation of the fundamental, and maximizes his expected profit. Hence the trading price  $p$  is equal to the expectation  $E[\theta]$ , which depends only on the public information at the time of the trade (including the trade). The market-maker does not know the type of a new customer but knows the structure of the model and computes rationally the equilibrium strategy of his customers. As emphasized by [Glosten and Milgrom \(1985\)](#), informed agents can trade and convey their information to the market because of the presence of asymmetric information between agent and market-maker.

In the second period, trade takes place in two steps. First, a new “young” agent comes to the market with a type determined as in the first period: with probability  $\alpha$ , he is exogenously informed, with probability  $\bar{\beta}$ , he is an information agent in which case he gets information with probability  $\beta_2/\bar{\beta}$ , and otherwise he is a noise trader. A trade in that stage has an impact on the price, as in the first period. Second, the “old” agent who traded in the first period cancels his position in the second period. This trade is identified by the market-maker as the resolution of a previous speculation and has no impact on the transaction price. (One may alternatively assume that agents contract the holding for one period with market-makers.) The value of  $\theta$  is revealed after the second period.

### 2.1 *The evolution of the price*

Assume first that the information investment in each period  $t$  ( $= 1, 2$ ), as measured by  $\beta_t$ , is given. Let  $x_t \in \{-1, 0, 1\}$  describe the event that in period  $t$ , the agent sells the asset, does not trade, or buys the asset. In a good state, an informed agent does not sell and an information agent who has no information does not trade because of the bid–ask spread. Hence, a sale can be generated only by a noise trader with probability  $\frac{1}{3}(1 - \alpha - \bar{\beta})$ . Similar arguments apply to the other cases. The probabilities of

transactions in the good state ( $\theta = 1$ ) and the bad state ( $\theta = 0$ ) are determined by

$$\begin{aligned} P(x_t = -1|\theta = 1) &= P(x_t = -1|\theta = 0) = \frac{1}{3}(1 - \alpha - \bar{\beta}) = \pi^0 \\ P(x_t = 1|\theta = 1) &= P(x_t = -1|\theta = 0) = \frac{1}{3}(1 - \alpha - \bar{\beta}) + \alpha + \beta_t = \pi^0 + \pi_t \\ P(x_t = 0|\theta = 1) &= P(x_t = 0|\theta = 0) = 1 - (2\pi^0 + \pi_t), \end{aligned} \quad (1)$$

with  $\pi_t = \alpha + \beta_t$ .

The variable  $\pi_t = \alpha + \beta_t$  measures the level of information (exogenous and endogenous) of an agent who comes to the market, and depends on the strategy  $\beta_t$ . This strategy depends only on the public information. Hence, the value of  $\pi_t$  is common knowledge and is used by the market-maker in the updating of the public belief after the observation of the transaction  $x_t$ . The value of  $\pi_t$  is also equal to the difference between the probabilities of a purchase and a sale, conditional on the “good” fundamental  $\theta = 1$ . (The case  $\theta = 0$  is symmetric.)

We may assume as a first step that  $\pi_t$  is a fixed parameter. (Its endogenous determination is analyzed in [Section 3](#).) The public belief at the beginning of period  $t$  is equal to the last period’s price  $p_{t-1}$  (or the last transaction price), with  $p_0 = \mu$  the initial belief. Let  $p^+(p_{t-1}, \pi_t)$  and  $p^-(p_{t-1}, \pi_t)$  be the values of the price in period  $t$  conditional on a buy ( $x_t = 1$ ) and a sale ( $x_t = -1$ ). Using Bayes’ rule and (1),

$$\begin{aligned} p^+(p_{t-1}, \pi_t) &= \frac{(\pi^0 + \pi_t)p_{t-1}}{(\pi^0 + \pi_t)p_{t-1} + \pi^0(1 - p_{t-1})} = \frac{(\pi^0 + \pi_t)p_{t-1}}{\pi^0 + \pi_t p_{t-1}} \\ p^-(p_{t-1}, \pi_t) &= \frac{\pi^0 p_{t-1}}{\pi^0 p_{t-1} + (\pi^0 + \pi_t)(1 - p_{t-1})} = \frac{\pi^0 p_{t-1}}{\pi^0 + \pi_t(1 - p_{t-1})}. \end{aligned} \quad (2)$$

The probability of no trade is the same when  $\theta = 1$  and  $\theta = 0$ . If there is no trade, there is no change in the public belief and by an abuse of notation, the price is the price of the last transaction. The difference between the ask  $p^+$  and the bid  $p^-$  is the spread:

$$\Delta(p_{t-1}, \pi_t) = p^+(p_{t-1}, \pi_t) - p^-(p_{t-1}, \pi_t) = \frac{p_{t-1}(1 - p_{t-1})\pi_t(\pi_t + 2\pi^0)}{(\pi^0 + \pi_t p_{t-1})(\pi^0 + \pi_t(1 - p_{t-1}))}. \quad (3)$$

The evolution of prices is presented in [Figure 1](#) for a case that is most relevant later: the public belief at the beginning of period  $t$  is high, possibly close to 1, and the true state is bad, contrary to the public belief. One should take  $t = 1$ . (The figure applies to any period in the infinite horizon model of the next section.) A sale is induced by an informed agent or noise trader with probability  $\pi^0 + \pi_t$  and is more likely than a buy, which is induced only by a noise trade with probability  $\pi^0$ .

## 2.2 The value of information

Consider an agent with a private belief  $v$  (which may be derived from public and private information), who trades in the first period and plans to liquidate his position in the next period. The price in the next period  $p_2$  is different from  $p_1$  if there is a transaction in period 2, in which case it depends on  $\pi_2$  according to the updating rule (2). Since the

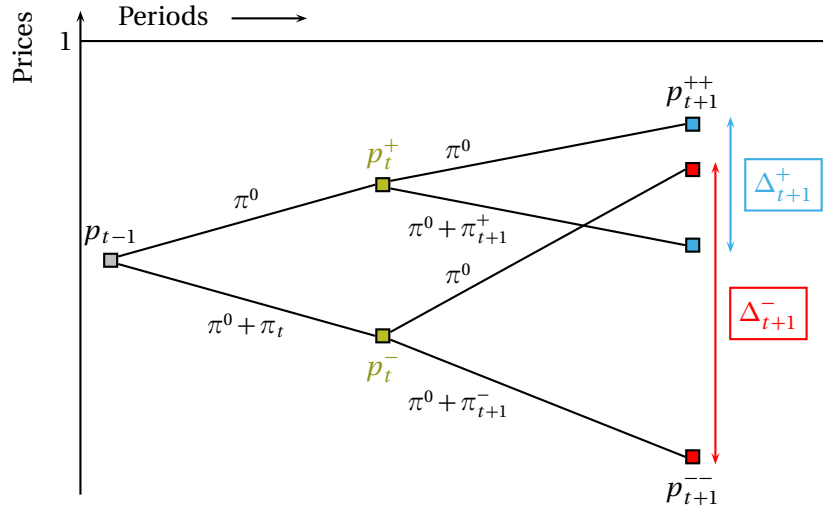


FIGURE 1. The evolution of the price. The true state is  $\theta = 0$  and the price is near one. (The case  $\theta = \theta_1$ , arbitrarily large and with a low belief, is symmetric.) For a description, see the text. The values of the prices  $p_t^+$ ,  $p_t^-$  and the probabilities  $\pi_t$  are common knowledge and can be computed before the game begins. In general,  $\pi_{t+1} = \alpha + \beta_{t+1}$  may depend on the history, hence the notation  $\pi_{t+1}^+$  and  $\pi_{t+1}^-$ . Movements along the branches are random and determined by the probabilities  $\pi^0, \pi^0 + \pi_t$ , etc.

probabilities of a purchase and a sale in period 2, conditional on the good state  $\theta = 1$ , are  $\pi^0 + \pi_2$  and  $\pi^0$  (and vice-versa if  $\theta = 0$ ), the value of holding one unit of the asset in period 1,  $u(v)$ , is the sum of the price  $p_1$  and the expected capital gain in each of the two states  $\theta = 1$  and  $\theta = 0$ , multiplied by their probabilities  $v$  and  $1 - v$ :

$$u(v) = p_1 + v((\pi^0 + \pi_2)(p^+(p_1, \pi_2) - p_1) + \pi^0(p^-(p_1, \pi_2) - p_1)) + (1 - v)(\pi^0(p^+(p_1, \pi_2) - p_1) + (\pi^0 + \pi_2)(p^-(p_1, \pi_2) - p_1)). \tag{4}$$

Because the price  $p_1$  satisfies the martingale property, we can write the same equation for the market-maker, who replaces  $v$  and  $u$  by  $p_1$ . That equation is subtracted from (4) and we have

$$u(v) - p_1 = \pi_2 \Delta(p_1, \pi_2)(v - p_1). \tag{5}$$

The absolute value of this expression represents the value of the optimal trade (buy one unit if  $u(v) - p_1 > 0$ , and sell one unit otherwise).

After the agent acquires information, his belief is either  $v = 1$ , in which case he buys at  $p_1^+$ , or  $v = 0$ , in which case he sells at  $p_1^-$ . The probability of his learning that  $\theta = 1$  is equal to  $\mu$ , the public belief at the beginning of period 1. The value of acquiring the information at the beginning of period 1 is therefore

$$V(\pi_1; \mu, \pi_2^+, \pi_2^-) = \mu(1 - p_1^+) \pi_2^+ \Delta(p_1^+, \pi_2^+) + (1 - \mu) p_1^- \pi_2^- \Delta(p_1^-, \pi_2^-). \tag{6}$$

A distinction is made here between  $\pi_2^+$  and  $\pi_2^-$ , which is the value of  $\pi_2$  after a purchase or a sale in period 1, in order to have a general expression for the next section, but can

be ignored here. The key effect appears immediately. *The value of information at the beginning of period 1 is an increasing function of the variability of the price in the next period following each of the two possible prices at the end of period 1.* That variability is measured by the spread in period 2 multiplied by the probability difference between the events of a buy and a sale, at the high or the low price, in that period. (See [Figure 1](#).)

### 2.3 Strategic complementarity of investment information within a period

The value of information in (6) depends on the ask and the bid in that period,  $p_1^+$  and  $p_1^-$ , which are functions of  $\pi_1 = \alpha + \beta_1$  in (2). The investment strategy  $\beta_1$  is public information in a rational expectation equilibrium and is used by the market-maker to set the bid and the ask. When the investment is higher, the market-maker raises the ask  $p_1^+$  and lowers the bid  $p_1^-$  in equation (2). This is the [Grossman and Stiglitz \(1980\)](#) effect. It reduces the value of information.

Now consider the impact on the variability of the price  $p_2$  as measured by the terms  $\pi_2^+ \Delta(p_1^+, \pi_2)$  and  $\pi_2^- \Delta(p_1^-, \pi_2^-)$ . Assume for the discussion that the “initial confidence is strong,” and that  $\mu = p_0$  is near 1, as in [Figure 1](#). (The case of  $\mu$  near 0 is symmetric.) A higher ask  $p_1^+$  in period 1 entails a higher confidence in the belief and therefore a lower impact of a transaction in the next period on  $p_2$ : the spread  $\Delta(p_1^+, \pi_2^+)$  is reduced. (Recall that  $\pi_2^+$  is exogenous in this section.) But if a sale takes place in period 1, the lower confidence at the beginning of period 2 generates more variability of  $p_2$  and a larger spread, which increases the value of information. When  $\mu = p_0$  is near 1 the two effects are not symmetric: an increase of confidence when confidence is already high reduces the value of information by a small amount. But news that reduces the confidence has an impact on the value of information that will be shown to dominate the combination of the first effect and the Grossman–Stiglitz effect of the previous paragraph.

Omitting the arguments  $\pi_2^+$  and  $\pi_2^-$ , which are fixed, and using (6), (3), and some manipulations, we find

$$V(\pi_1; \mu) = \kappa(\pi^0 + \pi_1) \left( \frac{1}{(\pi^0 + \pi_2 p_1^+)(\pi^0 + \pi_2(1 - p_1^+))(\pi^0 + \pi_1 \mu)^3} + \frac{1}{(\pi^0 + \pi_2 p_1^-)(\pi^0 + \pi_2(1 - p_1^-))(\pi^0 + \pi_1(1 - \mu))^3} \right),$$

with  $\kappa = (\pi_2)^2(\pi_2 + \pi^0)\mu^2(1 - \mu)^2(\pi^0)^2$ .

If  $\mu$  is near 1 or near 0, we can make the approximation

$$\frac{\partial V(\pi_1; \mu)}{\partial \pi_1} \approx A \left( 1 - 2 \left( \frac{\pi^0}{\pi^0 + \pi_1} \right)^3 \right),$$

where  $A > 0$  is a function of the parameters of the model that is independent of  $\pi_1$ . The right-hand side is positive if  $\pi^0(2^{1/3} - 1) < \pi_1$ . Recall that we must have  $\pi^0 + \pi_1 \leq 1$ . If  $\pi^0 < \frac{1}{2}$ , the interval  $(\pi^0(2^{1/3} - 1), 1 - \pi^0]$  is well defined. Since the previous expression is an approximation when  $\mu$  is near 0 or 1, we have the following result.



**PROPOSITION 1** (Strategic complementarity within a period). *If the noise parameter  $\pi^0$  is smaller than  $\frac{1}{2}$ , there exist  $\bar{\mu}$  and  $\bar{\pi}$  such that if the public belief  $\mu$  at the beginning of the first period is smaller than  $\bar{\mu}$  or greater than  $1 - \bar{\mu}$ , and the probability of an informed agent in the first period,  $\pi_1$ , is greater than  $\bar{\pi}$ , then the value of information is an increasing function of  $\pi_1$ .*

The upper-bound condition on noise trading is not very restrictive and is intuitive: if noise trading is large, the trades of the informed agents have little impact on the price. As the variability of the price becomes smaller, the incentive for a short-term trader to get information also becomes smaller. Under the conditions of **Proposition 1**, the payoff of anyone's investment in information increases in the investment made by others: there is strategic complementarity in getting information within the first period.

#### 2.4 Strategic complementarity between periods

A higher information investment in period 2 raises  $\pi_2$  and the variability of investment  $\pi_2 \Delta(p_1, \pi_2)$  for any  $p_1$ . This intuitive property is verified by simple algebra. The higher variability of the price in period 2 increases the value of information at the beginning of the previous period 1 (in expression (6)). One has the following result.

**PROPOSITION 2** (Strategic complementarity from a period to the previous one). *The value of information investment in period 1,  $V(p_0, \pi_1, \pi_2^+, \pi_2^-)$ , defined in (6), is increasing in the level of next period investment,  $\beta_2$ , and therefore increasing in  $\pi_2^+$  and  $\pi_2^-$ .*

The increasing value of information may generate multiple equilibria, a trivial property at this stage in the two-period model. After the exposition of the main mechanisms, we now consider the setting with an infinite number of periods with an endogenous information investment in each of them. **Proposition 2** shows that the equilibrium levels of investment in all periods are linked. This non-trivial problem is analyzed in the next section.

### 3. INFINITE HORIZON

The model is extended to an infinite number of periods, with a small probability of revelation of the fundamental in each period. This assumption is introduced to obtain a stationary solution. As before,  $\theta$  is set randomly before the first period and is constant through time. In each period,  $\theta$  is revealed with probability  $\delta$ , conditional on no previous revelation. The value of  $\delta$  is small, in a sense that is made more precise later.

If  $\theta$  is not revealed at the beginning of a period  $t$  (with probability  $1 - \delta$ ), trading takes place as in the simple model: a new agent is of one of the three types described in the previous section, acquires information at a fixed cost  $c$  if he can and finds it profitable, and meets a risk-neutral profit-maximizing market-maker to trade one unit of the asset or not to trade. The market-maker does not observe the type of the agent but has rational expectations and can compute the strategy of an information agent, which is based on public information.

After the new agent meets the market-maker, the “old” agent (who traded in the previous period), cancels his position and that trade has no impact on the price since it is identified as the profit (or loss) taking of a previous speculative trade. The risk-neutral market-maker trades for the long-term until the eventual revelation of  $\theta$ , or for the short-term (when he may unwind his position with another market-maker in the next period). Both assumptions are equivalent because of the law of iterated conditional expectations with rational agents.<sup>6</sup>

Since  $p_{t-1}$  summarizes the public information at the beginning of period  $t$ , the strategies of the information agents are assumed to be Markov strategies that are defined by measurable functions  $B_t(p_{t-1})$  from  $(0, 1)$  to the closed interval  $[0, \bar{\beta}]$ . Without loss of generality, all information agents follow the same strategy, which is common knowledge.

### 3.1 The evolution of the price and the value of information

If  $\theta$  is not revealed in period  $t$ , the trade  $x_t \in \{-1, 0, 1\}$  defines a public signal with the probabilities established in the equations (1) of the simple model. The ask and the bid are given by the Bayesian equations (2) in the previous section. The profit from a transaction  $x_t \in \{-1, 1\}$  by an agent with probability assessment  $v$  of  $\theta = 1$  is found as in (5) with an additional term for the possible revelation of the fundamental. It is the product of  $x_t$  and

$$\omega(v, p_t; \pi_{t+1}) - p_t = [(1 - \delta)\pi_{t+1}\Delta(p_t, \pi_{t+1}) + \delta](v - p_t).$$

Note the difference between the long-term and the short-term motive. If agents trade for the long-term,  $\delta = 1$ . We neglect this motive by taking  $\delta$  arbitrarily small. The gain from trade is not very different from the “pure” short-term gain in the first term which, as in the previous section, is the product of  $\pi_{t+1}$  and the spread  $\Delta(p_t, \pi_{t+1}) = p^+(p_t, \pi_{t+1}) - p^-(p_t, \pi_{t+1})$ .

Let  $\pi_{t+1}^+$  and  $\pi_{t+1}^-$  be the values of the probability  $\pi_{t+1}$  of an informed agent in period  $t + 1$  after a price increase (with a purchase) and a price decrease (with a sale) in period  $t$ . The value of information at the beginning of period  $t$  is found as in equation (6) of the simple model and is equal to

$$V(p_{t-1}, \pi_t, \pi_{t+1}^+, \pi_{t+1}^-) = (1 - \delta)\tilde{V}(p_{t-1}, \pi_t, \pi_{t+1}^+, \pi_{t+1}^-) + \delta L(p_{t-1}, \pi_t), \quad (7)$$

with

$$\begin{aligned} \tilde{V}(p_{t-1}, \pi_t, \pi_{t+1}^+, \pi_{t+1}^-) &= p_{t-1}(1 - p_t^+(p_{t-1}, \pi_t))\pi_{t+1}^+\Delta(p_t^+, \pi_{t+1}^+) \\ &\quad + (1 - p_{t-1})p_t^-(p_{t-1}, \pi_t)\pi_{t+1}^-\Delta(p_t^-, \pi_{t+1}^-) \end{aligned} \quad (8)$$

and

$$L(p_{t-1}, \pi_t) = p_{t-1}(1 - p_t^+) + (1 - p_{t-1})p_t^-.$$

<sup>6</sup> Let  $h_{t+1}$  be the history at the end of period  $t$ , i.e., the sequence of transactions including that of period  $t$ . If the market-maker holds the asset for one period, then  $p_t = \delta E[\theta|h_{t+1}] + (1 - \delta)E[p_{t+1}|h_{t+1}]$ . Taking the expected value of this equation in the next period,  $E[p_{t+1}|h_{t+1}] = E[\delta E[\theta|h_{t+2}] + (1 - \delta)E[p_{t+2}|h_{t+2}]|h_{t+1}] = \delta E[\theta|h_{t+1}] + (1 - \delta)E[p_{t+2}|h_{t+1}]$ . By iterations over all future periods,  $p_t = E[\theta|h_{t+1}]$ . This equation applies when the market-maker trades for the long term.

The expression  $\tilde{V}$  represents the expected payoff of information investment from short-term trading and is similar to the expression (6) in the simple model, while  $L$  represents the payoff from long-term trading (for an agent who waits for the revelation of the fundamental).

The value of  $V$  depends on  $\pi_{t+1}^+$  and  $\pi_{t+1}^-$  only through the function  $\tilde{V}$  and one can show, as in Proposition 2, that  $\tilde{V}$  is increasing in these two variables. It follows immediately that information investment in period  $t + 1$  increases the payoff of information investment at the beginning of period  $t$ .

**COROLLARY 1** (Strategic complementarity between periods). *The expected value of information investment in any period  $t$ ,  $V(p_{t-1}, \pi_t, \pi_{t+1}^+, \pi_{t+1}^-)$ , is an increasing function of  $\pi_{t+1}^+ = \alpha + \beta_{t+1}^+$  and  $\pi_{t+1}^- = \alpha + \beta_{t+1}^-$ .*

### 3.2 Symmetry

The model exhibits a symmetry between the high and the low values of the price  $p_{t-1}$  with respect to the middle price  $\frac{1}{2}$ . This symmetry is expressed by the following property of the value function:

$$V(p_{t-1}, \pi_t, \pi_{t+1}^+, \pi_{t+1}^-) = V(1 - p_{t-1}, \pi_t, \pi_{t+1}^-, \pi_{t+1}^+). \quad (9)$$

The analysis focuses on values of the price above  $\frac{1}{2}$ .

#### 3.3 The impact of the belief from history on the value of information

In the present setting, the uncertainty about the fundamental is measured by its variance  $p_{t-1}(1 - p_{t-1})$ . When  $p_{t-1}$  is greater than  $\frac{1}{2}$ , this uncertainty is a decreasing function of  $p_{t-1}$ . There is a positive relation between uncertainty and the value of information. This intuitive property is formalized in the next result, which is proved in the Appendix. Throughout the paper, increasing (decreasing) means strictly increasing (decreasing).

**LEMMA 1.** *For any probabilities  $(\pi_t, \pi_{t+1}^+, \pi_{t+1}^-) \in [\alpha, \alpha + \bar{\beta}]^3$ , the value of information  $V(p_{t-1}, \pi_t, \pi_{t+1}^+, \pi_{t+1}^-)$  defined in (7) is decreasing in  $p_{t-1}$  if  $p_{t-1} > \hat{p}$  (increasing if  $p_{t-1} < 1 - \hat{p}$ ), where  $\hat{p}$  is defined by  $\min_{\beta \in [0, \bar{\beta}]} p_t^-(\hat{p}, \alpha + \beta) = \frac{1}{2}$ .*

Because of the interactions between periods, an equilibrium strategy is defined as a sequence of investment probabilities. We identify such a strategy  $\beta_t/\bar{\beta}$  by the probability  $\pi_t = \alpha + \beta_t$  of an informed agent in period  $t$ . Any such sequence beginning in period  $t$  must depend on the history up to period  $t$ . It is reasonable to assume that this dependence is only on the public belief at the beginning of period  $t$ , which is equal to  $p_{t-1}$ . (Other previous prices could matter as coordination devices for an equilibrium, but such an assumption would be artificial.) We consider therefore the following class of equilibria.

**DEFINITION 1** (Equilibrium). *An equilibrium strategy is defined by a sequence of measurable functions  $\{B_t(p)\}_{t \geq 1}$  from  $(0, 1)$  to  $[0, \bar{\beta}]$  such that for any  $p \in (0, 1)$ ,*

- if  $B_t(p) = 0$ , then  $V(p, B_t(p), \pi_{t+1}^+, \pi_{t+1}^-) \leq c$
- if  $0 < B_t(p) < 1$ , then  $V(p, B_t(p), \pi_{t+1}^+, \pi_{t+1}^-) = c$
- if  $B_t(p) = \bar{\beta}$ , then  $V(p, B_t(p), \pi_{t+1}^+, \pi_{t+1}^-) \geq c$ ,

with  $\pi_{t+1}^+ = \alpha + B_{t+1}(p^+(p, B_t(p)))$ ,  $\pi_{t+1}^- = \alpha + B_{t+1}(p^-(p, B_t(p)))$ .

An equilibrium strategy  $B_t$  in period  $t$  depends on the strategy  $B_{t+1}$  in the next period. In general, the structure of equilibria is complex. We focus on stationary strategies where the function  $B_t$  does not depend on  $t$ ; this class is sufficiently rich, and, under imperfect information (Section 5), the unique equilibrium is a stationary strategy.

#### 4. STATIONARY EQUILIBRIUM STRATEGIES

We have seen in Lemma 1 that the information payoff is decreasing in  $p$  if  $p$  is high, and increasing if  $p$  is low. It is therefore natural to consider strategies in which an information agent invests if and only if the price is in some interval  $(p^{**}, p^*)$ .

DEFINITION 2 (Trigger strategy). A (stationary) *trigger strategy* is defined by an investment interval<sup>7</sup>  $(p^{**}, p^*)$  such that  $B(p) = 1$  if  $p \in (p^{**}, p^*)$ , and  $B(p) = 0$  if  $p \notin (p^{**}, p^*)$ .

Since there is a symmetry between the high and the low value of  $p$  (equation (9)), we focus on the determination of the upper-end of the investment interval,  $p^*$ . The value of information in period  $t$  depends on the level of information investment in period  $t + 1$ , which depends on the price  $p_t$ . Assume that  $p_{t-1}$  is near or at  $p^*$ . If the information agent learns that  $\theta = 1$ , he buys at the ask  $p_t^+$  which is above  $p^*$  and, by definition of the stationary strategy, there is no information investment in the next period. If he sells at the bid, the price  $p_t$  is in the investment interval and  $\beta_{t+1} = \bar{\beta}$ . We are thus led to introduce *zero-one expectations* such that  $\pi_{t+1}^+ = \alpha$  and  $\pi_{t+1}^- = \alpha + \bar{\beta}$ . Under zero-one expectations, in the period that follows a transaction at the ask (bid), no information agent (any information agent) invests. Using (7), omitting the time subscript with  $p_{t-1} \equiv p$ , and recalling  $\pi = \alpha + \beta$ , the payoff of investment under zero-one expectations is equal to

$$W(p, \beta) = V(p, \pi, \alpha, \alpha + \bar{\beta}) = (1 - \delta)\widetilde{W}(p, \beta) + \delta L(p, \pi), \quad (10)$$

with  $\widetilde{W}(p, \beta) = \widetilde{V}(p, \alpha + \beta, \alpha, \alpha + \bar{\beta})$ , defined in (8).

The expression  $\widetilde{W}(p, \beta)$  defines the payoff of information with pure short-term trade and zero-one expectations as a function of the last transaction price  $p$  and the information investment  $\beta$  in the current period. We first analyze the properties of this function. We later show that the component from long-term trade,  $\delta L(p, \pi)$ , can be neglected if  $\delta$  is sufficiently small. The next result shows that under some assumptions, the payoff from pure short-term trade generates strategic complementarities.

<sup>7</sup> The interval  $(p^{**}, p^*)$  is open, but one could include boundaries without altering the equilibrium since the price is at one of the boundaries with zero probability.

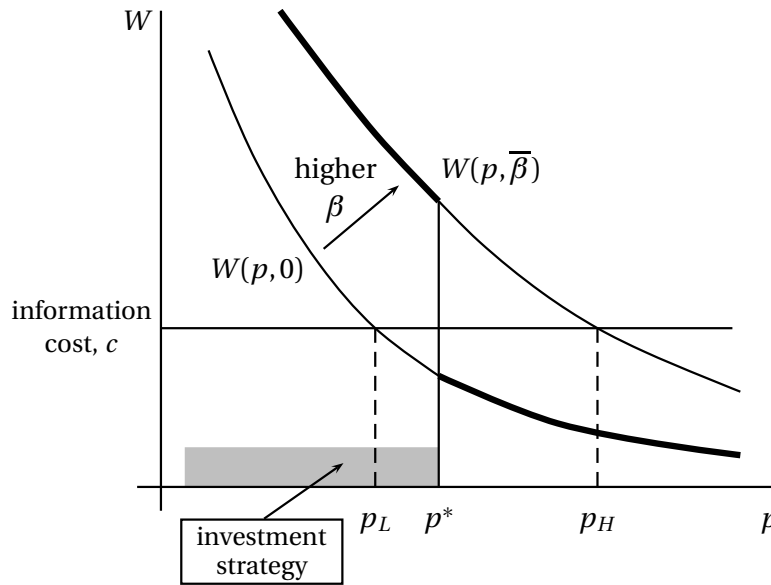


FIGURE 2. Continuum of constant equilibria. The function  $W(p, \beta)$  is the ex ante value of information for a last transaction price equal to  $p$ , a level of investment this period equal to  $\beta \in [0, \bar{\beta}]$  and investment equal to  $\bar{\beta}$  (0) if the price this period goes up (down).

PROPOSITION 3. For given  $\alpha$  and  $\bar{\beta}$  with  $\alpha < \bar{\beta}/3$ , there exists  $\bar{p}$  such that for any  $p \in (\bar{p}, 1)$ , the value of information with pure short-term trading and zero-one expectations,  $\widetilde{W}(p, \beta)$ , defined in (10), is decreasing in  $p$  and increasing in  $\beta$ .

The first part of the result is proved as Lemma 1. The second part, which is proved in the Appendix, holds only if the exogenous level of information  $\alpha$  is not too large relative to the range of values of the endogenous information,  $\bar{\beta}$ . Such an assumption is not surprising: a higher value of  $\alpha$  generates a higher ask and a lower bid by the rational market-maker, which reduces the payoff of information investment.

The properties of the function  $\widetilde{W}(p, \beta)$  in Proposition 3 are illustrated in Figure 2, where we can replace  $W$  by  $\widetilde{W}$ . From Proposition 3 and since for given  $\beta$ ,  $\widetilde{W}(p, \beta)$  is decreasing to 0 when  $p$  tends to 1, if  $c$  is not too high, the equation  $\widetilde{W}(p, 0) = c$  has a unique solution  $\tilde{p}_L$  such that  $\tilde{p}_L > \bar{p}$ , where  $\bar{p}$  is defined in Proposition 3. In this case, there is another solution  $\tilde{p}_H$  such that  $\widetilde{W}(\tilde{p}_H, \bar{\beta}) = c$  as represented in Figure 2 (where we can substitute  $\tilde{p}_L$  for  $p_L$  and  $\tilde{p}_H$  for  $p_H$ ). If the probability of revelation  $\delta$  is sufficiently small, then the function  $\widetilde{W}$  with pure short-term trading approximates arbitrarily closely the payoff of information  $W$  and Figure 2 applies to the graph of the function  $W(p, \beta)$ . This is the meaning of the next result.

PROPOSITION 4. Assuming  $\alpha < \bar{\beta}/3$ , there exists  $\bar{c}$  such that if  $c < \bar{c}$ , then there is  $\bar{\delta}$  such that if  $\delta < \bar{\delta}$ , for any  $\beta \in [0, \bar{\beta}]$ , the equation  $W(\phi(\beta), \beta) = c$  has a unique solution

$\phi(\beta) \in [\frac{1}{2}, 1]$ . Furthermore,

$$W(p, \beta) \begin{cases} > c & \text{if } p \in [\frac{1}{2}, \phi(\beta)) \\ < c & \text{if } p \in (\phi(\beta), 1), \end{cases}$$

$$\frac{\partial W(p, \beta)}{\partial \beta} > 0 \text{ if } p \in [p_L, p_H] \text{ with } W(p_L, 0) = W(p_H, \bar{\beta}) = c.$$

Choose a value  $p^* \in (p_L, p_H)$  as represented in the figure, and a value  $p^{**} \in (1 - p_H, 1 - p_L)$  (in the low range that is not represented). The next result shows that the trigger strategy  $(p^{**}, p^*)$  defines an equilibrium.

**PROPOSITION 5.** *Under the assumptions of Proposition 4, there is a continuum of stationary equilibrium strategies: any pair  $\{p^{**}, p^*\}$  such that  $p^* \in [p_L, p_H]$  and  $p^{**} \in [1 - p_H, 1 - p_L]$ , where  $\{p_L, p_H\}$  is defined in Proposition 4, defines a trigger strategy that is an equilibrium strategy.*

The sufficient conditions for the existence of a continuum of equilibrium strategies are simple: the cost of information should be smaller than some value, traders should sufficiently care about the short-term profits, and the occurrence of exogenously informed agents should not be too high compared to that of the traders for whom information is endogenous.

#### 4.1 Remarks

**4.1.1 The case of high information cost** When  $c$  is sufficiently large (but not too large), the solution  $\tilde{p}_L$  of  $\tilde{W}(\tilde{p}_L, 0) = c$  may be smaller than  $\bar{p}$  and Propositions 3 and 4 may not apply. In this case, there may be strategic substitutability and a unique equilibrium strategy. If  $p$  is greater than some value  $p^*$ , there is no information acquisition. If the price decreases from  $p^*$ , information investment increases gradually, possibly up to its maximum  $\bar{\beta}$ : the strategy for an information agent is to randomize with an increasing probability to acquire information as the price decreases. (If the price becomes lower than  $\frac{1}{2}$ , investment in information decreases.) The detailed analysis of this case is not the main focus in this paper and is left aside.

**4.1.2 Heterogeneous costs of information** We have assumed for simplicity that all agents have the same cost of information, but the equilibrium properties are robust when there is some cost heterogeneity. Suppose that an information agent can acquire information at the fixed cost  $c$ , which is an increasing function  $c(\beta)$  for  $\beta \in [0, \bar{\beta}]$ , with  $c(0) > 0$ . The distribution of costs is represented by a density function over  $\beta$  on the interval  $[0, \bar{\beta}]$ . An information agent is now characterized by a random draw from this distribution. If there is strategic complementarity, Proposition 4 holds with a minor alteration:  $p_L$  and  $p_H$  are defined by  $W(p_L, 0) = c(0)$  and  $W(p_H, \bar{\beta}) = c(\bar{\beta})$ . If there is strategic substitution, the equilibrium strategy becomes deterministic with a threshold value  $c^*$ . The observed investment in information is still random because of the random determination of the agent's cost.

**4.1.3 Convergence** The public belief is equal to the price of the asset and is a bounded martingale, hence it converges. If the probability of an exogenously informed agent  $\alpha$  is strictly positive, the price converges to the value of the fundamental. If there is no exogenously informed agent and  $\alpha = 0$ , the price does not converge to the true value because the value of information would tend to zero while the cost of information is strictly positive.

**4.1.4 Endogenous hedging** The assumption of exogenous noise traders can be replaced by the assumption of traders who hedge against an exogenous source of individual income that is correlated with the fundamental. Adapting the model of Dow (2004), one can assume that hedgers trade one unit of the asset and are differentiated by the marginal utility of income in the good and bad states. The cost of hedging increases with the bid–ask spread. When information agents buy more information, the widening of the spread crowds some hedgers out of trade. This effect increases the information content and the variability of the prices and therefore the value of information. It reinforces the strategic complementarity and the range of parameter values for a continuum of equilibria.<sup>8</sup>

## 5. IMPERFECT INFORMATION AND EQUILIBRIUM UNIQUENESS

The property of multiple equilibria is indicative of potentially large changes in the evolution of the price, but it leaves open the problem of coordinating on the strategy  $\{p^{**}, p^*\}$ , since there is a continuum of such values. Furthermore, one should check that the property is robust to a perturbation. In this section, we introduce an observation noise, which can be vanishingly small, on the history of prices. The game is then dominance solvable under some minor additional assumption specified below: there is a unique strategy that survives the iterated elimination of dominated strategies and is therefore a Strongly Rational-Expectations Equilibrium (SREE) (Guesnerie 2002). That strategy is one of the trigger strategies analyzed in the previous section.

The setting is similar to the one-period global game of Carlsson and Damme (1993), with a notable difference however: since the optimal strategy in any period  $t$  depends on the strategy in period  $t + 1$ , the eductive argument that eliminates strategies has to be applied backwards through time for all periods.

By assumption, the information agent who comes to the market in period  $t$  knows imperfectly the last transaction price  $p_{t-1}$ : his private information is the signal

$$s_t = p_{t-1} + \epsilon_t,$$

where  $\epsilon_t$  is independently drawn from a distribution with support  $[-\sigma, \sigma]$ . The analysis holds for any nondegenerate distribution of  $\epsilon_t$ , but to simplify  $\epsilon$  has a uniform distribution. The prior distribution on  $p_{t-1}$  is common knowledge and without loss of generality is assumed to be uniform.<sup>9</sup>

<sup>8</sup>In Dow (2004), the endogenous hedging may be sufficient to generate a discrete set of multiple equilibria.

<sup>9</sup>When  $\sigma$  is arbitrarily small, the density of the prior  $p_{t-1}$  is nearly uniform for a given  $s_t$ . The important

Market-makers have perfect information as befits their role. (A noisy observation on their part would probably not change the results.) The other parameters of the model are the same as in Section 4 without observation noise, and are assumed to satisfy the assumptions in Proposition 4.

If an information agent, after observing his signal  $s_t$ , does not invest in information, he stays out of the market and does not trade because of the bid–ask spread, as in the case with no observation noise. If he invests at the cost  $c$ , he learns the exact value of the fundamental  $\theta$ , and trades whatever the equilibrium prices. A strategy is now a measurable function of the signal  $s_t$  and the level of the investment  $\beta_t$  of any “other” information agent who may be called to the market in the same period.

The strategy  $\beta_t$  is rationally anticipated by the market-maker. Since he knows the price  $p_{t-1}$ , he can compute the distribution of private signals and using the probability of facing an informed agent, he sets the bid and ask to maximize his expected profit from trade, as in the previous sections. We do not restrict the strategy to be a trigger strategy, but the payoff with a trigger strategy is a useful tool. In a trigger strategy, an information agent invests if his signal is in some interval  $(\hat{s}', \hat{s})$ . We focus on the behavior of agents near the value  $\hat{s}$ , which is shown to be in the interval  $(p_L, p_H)$ . Without loss of generality, we may assume that  $\hat{s}' = 1 - \hat{s}$ , and the trigger strategy is defined by  $\hat{s}$ . For  $\sigma$  sufficiently small, the level of investment is equal to

$$\beta(p, \hat{s}) = \bar{\beta} \min\left(\max\left(\frac{\hat{p} + \sigma - s}{2\sigma}, 0\right), 1\right). \tag{11}$$

In the model with perfect information, we used the payoff function  $W(p, \beta)$  with the zero-one expectations that in the period after a price rise (at the ask), there is no information investment, whereas after a transaction at the bid, investment is at the maximum  $\bar{\beta}$ . A similar function plays an important role here.

An information agent with signal  $s$  has a subjective probability of state  $\theta = 1$  equal to  $\mu(s)$  and a density function  $\phi(p|s)$  on the last transaction price  $p$ . Assuming that he has zero-one expectations about the next period investment, and that the market-maker anticipates the strategy  $\hat{s}$ , by extension of (6), the payoff of information is equal to the function

$$J_\sigma(s, \hat{s}) = (1 - \delta)\tilde{J}_\sigma(s, \hat{s}) + \delta K_\sigma(s, \hat{s}), \tag{12}$$

with

$$\begin{aligned} \tilde{J}_\sigma(s, \hat{s}) &= \mu(s) \int_{s-\sigma}^{s+\sigma} (1 - p^+) \pi^+ \Delta(p^+, \pi^+) \phi(p|s) dp \\ &\quad + (1 - \mu(s)) \int_{s-\sigma}^{s+\sigma} p^- \pi^- \Delta(p^-, \pi^-) \phi(p|s) dp \\ K_\sigma(s, \hat{s}) &= \int_{s-\sigma}^{s+\sigma} (\mu(s)(1 - p^+) + (1 - \mu(s))p^-) \phi(p|s) dp, \end{aligned}$$

---

assumption is that the prior of an information agent has a support that includes an open interval that includes the interval  $[1 - p_H, p_H]$ .



where  $p^+ = p^+(p, \alpha + \beta(p, \hat{s}))$ ,  $p^- = p^-(p, \alpha + \beta(p, \hat{s}))$ ,  $\pi^+ = \alpha$ ,  $\pi^- = \alpha + \bar{\beta}$ , and  $\beta(p, \hat{s})$  is given by (11). The functions  $\tilde{J}_\sigma(s, \hat{s})$  and  $K_\sigma(s, \hat{s})$  are continuous and have continuous partial derivatives.

Using the uniform distributions of  $p_{t-1}$  and the signal  $s$ , the previous expressions can be rewritten

$$\begin{aligned} \tilde{J}_\sigma(s, \hat{s}) &= s \frac{1}{2\sigma} \int_{s-\sigma}^{s+\sigma} (1-p^+) \alpha \Delta(p^+, \alpha) dp \\ &\quad + (1-s) \frac{1}{2\sigma} \int_{s-\sigma}^{s+\sigma} p^-(\alpha + \bar{\beta}) \Delta(p^-, \alpha + \bar{\beta}) dp \quad (13) \\ K_\sigma(s, \hat{s}) &= s \frac{1}{2\sigma} \int_{s-\sigma}^{s+\sigma} (1-p^+) dp + (1-s) \frac{1}{2\sigma} \int_{s-\sigma}^{s+\sigma} p^- dp. \end{aligned}$$

### 5.1 Vanishingly small observation noise

We have to consider the case  $\hat{s} = s$  in an equilibrium. After some elementary manipulations,<sup>10</sup> we find

$$\lim_{\sigma \rightarrow 0} J_\sigma(s, s) = \frac{1}{\bar{\beta}} \int_0^{\bar{\beta}} W(s, \beta) d\beta, \quad (14)$$

where  $W(s, \beta)$  is defined in (10). Using the differentiability of the Bayesian functions  $p^+$  and  $p^-$  on  $[0, 1]$ ,

$$\lim_{\sigma \rightarrow 0} \frac{dJ_\sigma(s, s)}{ds} = \frac{1}{\bar{\beta}} \int_0^{\bar{\beta}} \frac{\partial W(s, \beta)}{\partial s} d\beta.$$

These equations show that for  $\sigma$  arbitrarily small, the function  $J_\sigma(s, s)$  is approximated by an average of the functions  $W(s, \beta)$ . The next result follows from the properties of  $W(p, \beta)$  in Propositions 4 and 5.

**LEMMA 2.** *Under the assumptions of Proposition 4, there exists  $\hat{\sigma}$  such that if  $\sigma < \hat{\sigma}$ , the equation  $J_\sigma(s, s) = c$  has a unique solution  $s^*$  on the interval  $[p_L, 1]$ . Furthermore,  $s^* \in (p_L, p_H)$ ,  $J_\sigma(s, s) < c$  for  $s > s^*$ , and if  $\sigma \rightarrow 0$  then  $s^* \rightarrow S^*$  which is defined by*

$$\frac{1}{\bar{\beta}} \int_0^{\bar{\beta}} W(S^*, \beta) d\beta = c.$$

<sup>10</sup>Using  $p = s + \sigma(2\beta/\bar{\beta} - 1)$  from (11), we have

$$\tilde{J}_\sigma(s, s) = \frac{s}{\bar{\beta}} \int_0^{\bar{\beta}} (1-p^+) \alpha \Delta(p^+, \alpha) d\beta + \frac{1-s}{\bar{\beta}} \int_0^{\bar{\beta}} p^-(\alpha + \bar{\beta}) \Delta(p^-, \alpha + \bar{\beta}) d\beta,$$

with

$$p^+ = p^+ \left( s + \sigma \left( 2 \frac{\beta}{\bar{\beta}} - 1 \right), \beta \right), \quad p^- = p^- \left( s + \sigma \left( 2 \frac{\beta}{\bar{\beta}} - 1 \right), \beta \right).$$

Recall that with perfect information, the short-term payoff of information (with  $\delta \approx 0$ ) is given in (10) and (8):

$$\tilde{W}(p, \beta) = p(1 - p^+(p, \pi)) \alpha \Delta(p^+, \alpha) + (1 - p) p^-(p, \pi) (\alpha + \bar{\beta}) \Delta(p^-, \alpha + \bar{\beta}),$$

with  $\pi = \alpha + \beta$ . Equation (14) follows with the expression of  $J_\sigma$  in (12) and  $K_\sigma$  in (13).

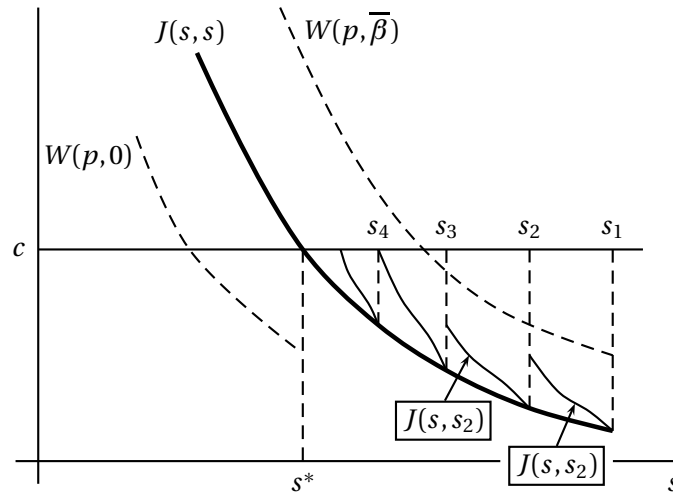


FIGURE 3. The value of information  $J(s, s)$  at the threshold  $s$  of a trigger strategy, and iterated dominance.  $J(s, s^*)$  is the value of information for an agent with signal  $s$  when other agents invest only if their signals are greater than  $s^*$ . For a discussion, see the proof of Proposition 6 in the Appendix.

The function  $J_\sigma(s, s)$  replaces the function  $W(p, \beta)$  that was used with perfect information. It is represented in Figure 3, which illustrates the following iterative dominance argument.

In the first step, there is a value of the private signal  $s_1$  such that if  $s > s_1$ , the agent is sufficiently confident that the state is good and the value of information is below the cost  $c$  even if all other agents invest. Investment is dominated, which implies that  $J(s_1, s_1) < c$ . When agents with a signal higher than  $s_1$  do not invest in period  $T$ ,  $T$  arbitrary, the value of investment in period  $T - 1$  is bounded above by the value under zero-one expectations, which is  $J(s, s_1)$ . By continuity of  $J$ ,  $J(s, s_1) < c$  on an interval  $(s_2, s_1]$ . Hence, investment is dominated in period  $T - 1$  for  $s < s_2$  if it is dominated in period  $T$  for  $s > s_1$ . The argument is used iteratively for the following result, which is proved in the Appendix.

PROPOSITION 6. Assume  $c$  and  $\delta$  such that Proposition 4 holds, and  $\alpha > 0$ . There exists  $\bar{\sigma}$  such that if  $\sigma < \bar{\sigma}$ ,

- (i) investment is iteratively dominated for any  $s > s^*$  where  $s^* \in (p_L, p_H)$  is defined by  $J_\sigma(s^*, s^*) = c$ . If  $\sigma \rightarrow 0$ , then  $s^* \rightarrow S^*$  such that

$$\frac{1}{\beta} \int_0^{\bar{\beta}} W(S^*, \beta) d\beta = c$$

- (ii) there exists a value  $\hat{c} \leq \bar{c}$  (defined in Proposition 3), such that if  $c < \hat{c}$  and  $\sigma < \bar{\sigma}$ , then for  $s \in (1 - s^*, s^*)$ , not to invest in information is iteratively dominated. In this

*case the strategy to invest if and only if  $s \in (1 - s^*, s^*)$  is a SREE. (It is the only strategy to survive the iterated elimination of dominated strategies.)*

The proposition introduces two minor assumptions: the restriction  $\alpha > 0$  ensures that the variation of the price after a transaction has a strictly positive lower bound. This lower bound ensures that zero-one expectations apply to an agent near a threshold value  $s^*$ : if he buys (sells), he is sure that for  $\sigma$  sufficiently small, all information agents in the next period will have a signal strictly higher (lower) than  $s^*$  and by definition of  $s^*$  will not invest (will invest) in information. The restriction on the cost  $c$  in Part (ii) is used to ensure that information investment is dominant if the price is near  $\frac{1}{2}$ . This restriction can be removed if we assume that agents use a trigger strategy in some period.

## 6. CONCLUSION

We began by showing that the combination of short-term trades and endogenous information generates strategic complementarities, and that these complementarities may be sufficiently strong to generate a continuum of equilibria when agents have common knowledge of the last transaction price. Each equilibrium defines regions of prices with sharply different levels of investment. When the price crosses a threshold, the level of investment is discontinuous because of the strategic complementarity.

Under small imperfect information, the continuum is reduced to a singleton, but there is no contradiction between the results without observation noise. The important property is the discontinuity of the information investment. The multiplicity of equilibria is not robust to a perturbation with observation noise, but the discontinuity in behavior is robust.

Take an initial price near 1 and a fundamental equal to 0. Eventually, the price must decrease (since it converges to the truth). When agents observe perfectly the history, the price enters a region with multiple equilibrium strategies with zero or maximum investment. When the observation of history is subject to a noise, however small, **Proposition 6** states that the level of investment grows from zero to its maximum when the price crosses the interval  $[S^* - \hat{\sigma}, S^* + \hat{\sigma}]$ , which is arbitrarily small.

There is a linear relation in the model between the probability of a trade and the level of endogenous information. Hence, in the equilibrium there is a positive relation between the volume of trade and the information that is generated by the market. Information frenzy is equivalent to trade frenzy.

The mechanism presented here is robust for other models of micro-structure, as verified numerically in the **Appendix** for standard model with a continuum of agents who place simultaneous limit orders in the same period.

A next step is to consider small random changes of the fundamental.<sup>11</sup> One may anticipate that the present results will be extended and that in an equilibrium, there will be random switches between two regimes with sharply different levels of trade and information.

<sup>11</sup>David (1997) analyzes the learning process in a financial market when the state switches randomly between discrete values and agents have exogenous private information.

APPENDIX

A.1 Proofs

PROOF OF LEMMA 1. The value function  $V$  is defined in (7), which is repeated here:

$$V = (1 - \delta)\tilde{V} + \delta [p_{t-1}(1 - p_t^+) + (1 - p_{t-1})p_t^-].$$

We omit the time subscripts when there is no ambiguity. In the second term of this expression,  $p_t^+$  and  $p_t^-$  are given by the Bayesian equations (2), and simple algebra shows that this term is a decreasing function of  $p_{t-1}$  if  $p_{t-1} > \frac{1}{2}$ . Focusing now on the first term, from (8),

$$\tilde{V}(p_{t-1}, \pi) = p_{t-1}(1 - p_t^+)\pi_{t+1}^+\Delta(p_t^+, \pi_{t+1}^+) + (1 - p_{t-1})p_t^-\pi_{t+1}^-\Delta(p_t^-, \pi_{t+1}^-).$$

The prices  $p_t^+$  and  $p_t^-$  are increasing in  $p_{t-1}$ . Since  $p_t^- > \frac{1}{2}$  under the assumption in the lemma, the spreads in period  $t + 1$ ,  $\Delta(p_t^+, \pi_{t+1}^+)$  and  $\Delta(p_t^-, \pi_{t+1}^-)$ , are decreasing in  $p_{t-1}$ .

A small exercise shows that  $p_{t-1}(1 - p_t^+)$  is decreasing in  $p_{t-1}$  if  $p_{t-1} > \frac{1}{2}$  and that  $(1 - p_{t-1})p_t^-$  is decreasing in  $p_{t-1}$  if  $p_t^- > 1 - p_{t-1}$ , which is satisfied by the assumption of the lemma ( $p_t^- > \frac{1}{2} > 1 - p_{t-1}$ ). The last part of the lemma follows from the symmetry in (9).  $\square$

PROOF OF PROPOSITION 3. The first part of the proposition was proved in Lemma 2. Recall the definition of  $W(p, \beta)$  in (10):

$$W(p, \beta) = (1 - \delta)\tilde{W}(p, \pi) + \delta L(p, \pi) \quad \text{with } \pi = \alpha + \beta.$$

We first prove that the short-term component of the information value exhibits strategic complementarity ( $\partial \tilde{W}(p, \pi) / \partial \pi > 0$ ), and then show that the long-term component can be ignored if  $\delta$  is sufficiently small. The short-term component is given in (8) for any period  $t$  and is equal to

$$\tilde{V}_t = p_{t-1} \frac{p_t^+(1 - p_t^+)^2(\pi_{t+1}^+)(\pi_{t+1}^+ + 2\pi^0)}{D_1(p_t^+, \pi_{t+1}^+)D_2(p_t^+, \pi_{t+1}^+)} + (1 - p_{t-1}) \frac{(p_t^-)^2(1 - p_t^-)(\pi_{t+1}^-)(\pi_{t+1}^- + 2\pi^0)}{D_1(p_t^-, \pi_{t+1}^-)D_2(p_t^-, \pi_{t+1}^-)}.$$

Using (2) and setting  $\pi = \alpha + \beta$ ,  $\pi_{t+1}^+ = \alpha$ , and  $\pi_{t+1}^- = \alpha + \bar{\beta}$  because of the definition of  $W(p, \beta)$ , replace  $p_{t-1}$  by  $p$ , and omit the time subscript because there is no ambiguity:

$$\tilde{W}(p, \pi) = p^2(1 - p)^2(\pi^0)^2 \mathcal{H}(p, \pi), \tag{15}$$

with

$$\mathcal{H}(p, \pi) = \frac{(\pi^0 + \pi)\alpha(\alpha + 2\pi^0)}{D_1^3(p, \pi)D_1(p^+, \alpha)D_2(p^+, \alpha)} + \frac{(\pi^0 + \pi)(\alpha + \bar{\beta})(\alpha + \bar{\beta} + 2\pi^0)}{D_2^3(p, \pi)D_1(p^-, \alpha + \bar{\beta})D_2(p^-, \alpha + \bar{\beta})}, \tag{16}$$

where  $D_1(p, \pi) = \pi^0 + \pi p$  and  $D_2 = \pi^0 + \pi(1 - p)$  are the denominators in (2).

Let  $a$  be the first term in (16). Its derivative with respect to  $\pi$  is

$$a'_\pi = a \left( \frac{1}{\pi^0 + \pi} - \frac{3p}{\pi^0 + \pi p} - \frac{\partial p^+}{\partial \pi} \left( \frac{\partial D_1}{\partial p^+} \frac{1}{D_1} + \frac{\partial D_2}{\partial p^+} \frac{1}{D_2} \right) \right).$$

This expression is negative if  $p > \frac{1}{2}$ , as befits the intuition: good news in period  $t$  increases the level of confidence and decreases the variability of the price in the next period. Taking the limit as  $p \rightarrow 1$  and using  $D_1(p, \pi) \rightarrow \pi^0 + \pi$  and  $D_2(p, \pi) \rightarrow \pi^0$ ,

$$a'_\pi \rightarrow -\frac{2\alpha(\alpha + 2\pi^0)}{(\pi^0 + \pi)^3(\pi^0 + \alpha)\pi^0}.$$

Likewise for the second term  $b$  in (16),

$$b'_\pi = b \left( \frac{1}{\pi^0 + \pi} - \frac{3(1-p)}{\pi^0 + \pi(1-p)} - \frac{\partial p^-}{\partial \pi} \left( \frac{\partial D_1}{\partial p^-} \frac{1}{D_1} + \frac{\partial D_2}{\partial p^-} \frac{1}{D_2} \right) \right) \rightarrow \frac{(\alpha + \bar{\beta})(\alpha + \bar{\beta} + 2\pi^0)}{(\pi^0)^4(\pi^0 + \alpha + \bar{\beta})}.$$

Combining the two previous expressions, if  $p$  tends to 1, then  $\mathcal{H}'_\pi(p, \pi) = a'_\pi + b'_\pi$  tends uniformly with respect to  $\pi$  to a limit  $\lambda(\pi)$ . Since  $\pi \geq \alpha$ ,

$$\lambda(\pi) > \frac{(\alpha + \bar{\beta})(\alpha + \bar{\beta} + 2\pi^0)}{(\pi^0)^4(\pi^0 + \alpha + \bar{\beta})} - \frac{2\alpha(\alpha + 2\pi^0)}{(\pi^0 + \alpha)^4\pi^0}.$$

If  $\alpha < \bar{\beta}/3$ , there exists  $\lambda_0 > 0$ . Hence, there exists  $\bar{p}$  such that if  $p > \bar{p}$ , then for all  $p > \bar{p}$ ,  $\mathcal{H}'_\pi > \lambda_0/2 > 0$ . Because of (15), a similar inequality applies to  $\widetilde{W}'_\pi$ .  $\square$

**PROOF OF PROPOSITION 4.** From the text that precedes Proposition 4, we have the following result.

**LEMMA 3.** *There exists  $\bar{c}$  such that if  $c < \bar{c}$ , then the equation  $\widetilde{W}(\tilde{\phi}(\beta), \beta) = c$  has a unique solution for  $\beta \in [0, \bar{\beta}]$ ,  $\tilde{\phi}(\beta) \in [\frac{1}{2}, 1]$ , and we have the following properties:*

- (i)  $\widetilde{W}(p, \beta) > c$  for  $p \in [\frac{1}{2}, \tilde{\phi}(\beta))$ ,  $\widetilde{W}(p, \beta) < c$  for  $p \in (\tilde{\phi}(\beta), 1)$ .
- (ii)  $\frac{\partial \widetilde{W}(p, \beta)}{\partial \beta} > 0$  if  $p \geq \tilde{\phi}(0)$ .

Define  $\tilde{p}_L = \phi(0)$ , and  $\tilde{p}_H = \tilde{\phi}(\bar{\beta})$ . The functions  $\widetilde{W}(p, \beta)$  and  $L(p, \beta)$  and their derivatives are continuous for  $(p, \beta) \in (\frac{1}{2}, 1) \times [0, \bar{\beta}]$ , therefore uniformly continuous on any compact subset of  $(\frac{1}{2}, 1) \times [0, \bar{\beta}]$ . There is an open interval  $(\hat{p}', \hat{p})$  containing  $[\tilde{p}_L, \tilde{p}_H]$  such that following Proposition 3,  $\partial \widetilde{W}(p, \beta)/\partial p$  has a strictly negative upper bound and  $\partial \widetilde{W}(p, \beta)/\partial \beta$  has a strictly positive lower bound for  $\hat{p}' \leq p \leq \hat{p}$  and  $0 \leq \beta \leq \bar{\beta}$ . One can choose  $\bar{\delta}$  such that (i) applies to the function  $W$  and (ii) applies also if  $p$  is not in an arbitrarily small interval that contains 1.  $\square$

**PROOF OF PROPOSITION 5.** Suppose first  $p_{t-1} > p^*$ . By definition of the strategy  $p^*$ , no information agent invests in period  $t$ . Consider the payoff of a deviating agent who invests. If after paying the cost  $c$ , he learns that  $\theta = 1$ , then he buys at  $p_t^+ > p_{t-1} > p^*$ . By the definition of the strategy  $p^*$ , no agent invests in the next period and  $\pi_{t+1}^+ = 0$ . We do not need to be concerned by the outcome if he learns that  $\theta = 0$  because of the strategic complementarity from period  $t+1$  to period  $t$ : from **Proposition 2**, the payoff of investment in period  $t$  is not greater than if  $\beta_{t+1} = \bar{\beta}$ . The payoff of investment in period  $t$  is therefore bounded above by the payoff under zero-one expectations,  $W(p_{t-1}, 0) < W(p_L, 0) = c$ , using **Proposition 4** and the definition of  $p_L$  in that proposition.

Suppose now that  $\frac{1}{2} \leq p_{t-1} < p^*$ : any information agent invests in period  $t$ . We consider again a deviating agent who invests. If he learns that  $\theta = 0$ , he trades at the bid  $p^-(p_{t-1}, \bar{\beta})$ . Using the property of  $\bar{p}$ ,  $\frac{1}{2} < p^-(p_{t-1}, \bar{\beta})$  and  $\pi_{t+1}^- = \alpha + \bar{\beta}$ . Again using **Corollary 1**, the payoff of investment in period  $t$  is now bounded below by the payoff under zero-one expectations,  $W(p, \beta)$ , which is strictly greater than  $c$ .  $\square$

**PROOF OF PROPOSITION 6.** To prove the result, one begins with a region of dominance. If  $s$  is sufficiently close to 1, for small  $\sigma$  the agent is sure that the price is close to 1 and the value of information is lower than the cost. There is a value  $s_1$  such that for any agent with a signal  $s \in (s_1, 1)$ , investment is dominated. The interval  $(s_1, 1)$  is now extended by iterations to the left.

A critical step in the case of perfect information is the use of zero-one expectations about the next period investment. Under imperfect information, an agent cannot be sure about the transaction price and therefore the distribution of agents in the next period. But under the trivial assumption  $\alpha > 0$ , if the heterogeneity is sufficiently small, which is the desirable case, it is dwarfed by the price movement and an individual with signal  $s$  is sure that if he makes a transaction, any information agent in the next period will be “to the right” or “to the left” by a quantum step. This is the meaning of the following lemma, which is easy to establish,<sup>12</sup> and which is illustrated by **Figure 4**.

**LEMMA 4.** *If  $\alpha > 0$ , there exist  $\hat{\sigma}_1$  and  $\eta > 0$  such that if  $\sigma < \hat{\sigma}_1$ , then for any  $s_1$  and  $s$  with  $\frac{1}{2} < s_0 < 1$ , and  $s \in (1 - s_1, s_1)$ , an agent with signal  $s$  is sure that if he buys (sells), the signal of any agent in the next period will be greater than  $s + \eta$  (smaller than  $s - \eta$ ).*

Choose an arbitrary period  $T$  (which is large in a sense defined below). In that period, information investment is dominated for  $s \in (s_1, 1)$ . Consider now an agent in period  $T - 1$  with a signal  $s$  in a neighborhood of  $s_1$ , as illustrated in **Figure 4**. By **Lemma 4**, that agent is sure that if he buys, any information agent in the next period has a signal higher than  $s_1$  and therefore does not invest, by definition of  $s_1$ . If he sells, the price falls, investment in the next period may not be known, but we can use the strategic complementarity from period  $T$  to period  $T - 1$  (**Corollary 1**) to put an upper bound on his value of information, which is therefore not greater than under zero-one expectations (zero investment next period if the agent gets a good signal and buys, maximum investment if he sells). The payoff of information is bounded above by the function  $J_\sigma(s, s_1)$ .

<sup>12</sup> Choose  $\hat{\sigma}_1 < \gamma/3$  and  $\eta < \gamma/3$ , where  $\gamma$  is the lower bound of the price change when  $s \in [1 - s_1, s_1]$ .

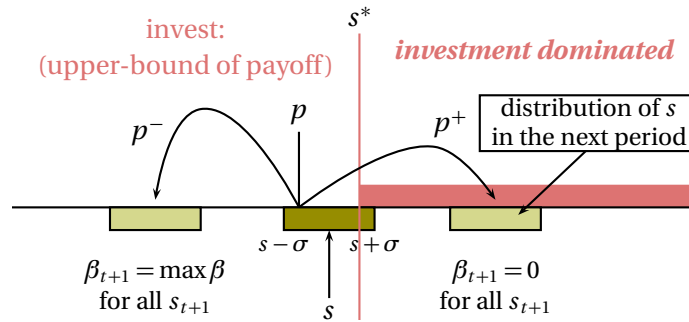


FIGURE 4. Iterated elimination of dominated strategies. While  $s$  moves a little to the left from  $s^*$ , the levels of investment in the next period do not change.

Since  $J_\sigma$  is continuous and  $J_\sigma(s_1, s_1) < c$ , there is a value  $s_2$  such that for any  $s \in (s_2, s_1)$ ,  $J_\sigma(s, s_1) < c$  and  $s_2 > s_1 - \eta$ , where  $\eta$  is defined in Lemma 4. The argument is illustrated in Figure 3. After this first step, investment is dominated for an agent with signal  $s \in (s_2, s_1]$  in period  $T - 1$ . Because investment is dominated for  $s \in (s_1, 1)$  for any period, it is dominated for  $s \in (s_2, 1)$  in period  $T - 1$  and therefore in any period (since  $T$  is arbitrary).

Repeating the argument for any period  $T - k \geq 1$  and taking  $T$  arbitrarily large, we construct a sequence  $\{s_k\}_{k \geq 1}$  that is decreasing,<sup>13</sup> bounded below by  $s^*$ , and thus converging to some  $\hat{s}^* \geq s^*$ . If  $\hat{s}^* > s^*$ , a small exercise that uses the continuity of  $J_\sigma(s, s)$  shows that the differences  $s_{k-1} - s_k$  are bounded below by a strictly positive number, which yields a contradiction. Hence, for any  $s > s^*$ , the information investment is dominated after a finite number of iterations. This proves Part (i).

For Part (ii), we need to find an interval of “medium values” of the private signals such that information investment is a dominating strategy. Define  $M$  such that

$$M(p) = \min_{\beta \in [0, \bar{\beta}]} (V(p, \alpha + \beta, \alpha, \alpha)).$$

Because of the continuity of the function  $V$ , which is increasing with respect to its last two arguments (Corollary 1),  $M(\frac{1}{2}) > 0$ .

There exists  $\bar{c}_1$  such that  $\bar{c}_1 < M(p)$  for  $p \in (\frac{1}{2} - \eta', \frac{1}{2} + \eta')$  for some value of  $\eta' > 0$ . Take  $\hat{c}$  in Proposition 6 to be the minimum of  $\bar{c}_1$  and the value  $\bar{c}$  defined in Proposition 4. Then there exists  $\underline{s}_0 > \frac{1}{2}$  such that an agent with signal  $s \in [1 - \underline{s}_1, \underline{s}_1]$  is sure that the value of information is higher than the cost  $c$ , independently of the strategies of others. One then uses an iterative argument as in Part (i) to generate an increasing sequence  $\{\underline{s}_k\}_{k \geq 1}$  that converges to  $s^*$ , such that investment is a dominating strategy for any interval  $(1 - \underline{s}_k, \underline{s}_k)$ . □

<sup>13</sup>In Figure 3,  $s_1 - s_2 = s_2 - s_3 = \eta$  but  $s_4 > s_3 - \eta$  and  $J_\sigma(s_4, s_3) = c$ .

### A.2 A model with simultaneous trading by a continuum of agents

In order to show that the increasing value of information holds in a standard structure with a continuum of agents, a two-period model is sufficient. As in [Section 2](#), there are two possible values of the fundamental,  $\theta_0 = 0$  and  $\theta_1 = 1$ . An informed agent receives a signal  $s_i = \theta + \epsilon$ , where  $\epsilon \sim \mathcal{N}(0, \sigma^2)$ . He is risk-neutral but his trade is limited in absolute value by a bound, which is normalized to one. The mass of informed traders in period 1 is  $\lambda_1 = \lambda$ , which is variable, and in period 2, with new agents,  $\lambda_2 = \bar{\lambda}$ , which is fixed.

In any period  $t$ , the demand is the sum of the demand by newly informed agents, noise traders,  $Q_t$  which is normally distributed  $\mathcal{N}(0, \omega^2)$ , and market-makers, as in the standard model of a financial market with a continuum of agents. Market-makers are risk-neutral, hence the equilibrium price is also the probability assessment of the state  $\theta = 1$ . In a rational expectation equilibrium, the observation of the market price is equivalent to the observation of the demand of informed agents plus that of the noise traders,  $Y_t = \lambda_t X_t(\theta) + Q_t$ , where  $X_t(\theta)$  is the average demand of informed agents. The realization  $Y_t$  provides a signal of  $\theta$ . As in [Section 2](#), the informed traders of period 1 cancel their positions after the equilibrium of period 2, with no impact on the price  $p_2$ , before the revelation of the fundamental.

The task is to determine the value of acquiring the signal  $s$  in the first period given the initial belief,  $\mu_0$ . This value  $V(\lambda_1)$  depends on the mass of informed agents in period 1.

An agent who has the signal  $s$  also observes the market price. He is more optimistic than the market if his signal is good, that is if  $s > s^* = (\theta_0 + \theta_1)/2 = \bar{\theta}$ . An exercise shows that our risk-neutral agent who is constrained on his trade and holds his position only for one period, buys (sells) up to the constraint when he is more (less) optimistic than the market: he buys if and only if  $s > s^*$ . The average net demand per informed agent is therefore

$$X(s^*; \theta) = 1 - 2F\left(\frac{s^* - \theta}{\sigma}\right).$$

Given the value of  $s^*$ , the demand is

$$X(\theta) = \text{sign}\left(\theta - \frac{\theta_1 + \theta_0}{2}\right) \bar{X}, \quad \text{with} \quad \bar{X} = 1 - 2F\left(\frac{\theta_0 - \theta_1}{2\sigma}\right).$$

The demand per informed agent is constant in absolute value, positive if  $\theta = 1$ .

Given the form of  $X(\theta)$ , the market signal  $Y_t = \lambda_t X(\theta) + Q_t$  can be replaced by  $Z_t = \lambda_t \theta + Q_t$ , where  $Q_t$  has a normal distribution  $\mathcal{N}(0, \omega^2)$ . This variable is itself observationally equivalent to  $\theta + Q_t/\lambda_t$ . In this form, one sees immediately that the signal to noise ratio that is provided by the market increases with the proportion of informed agents.



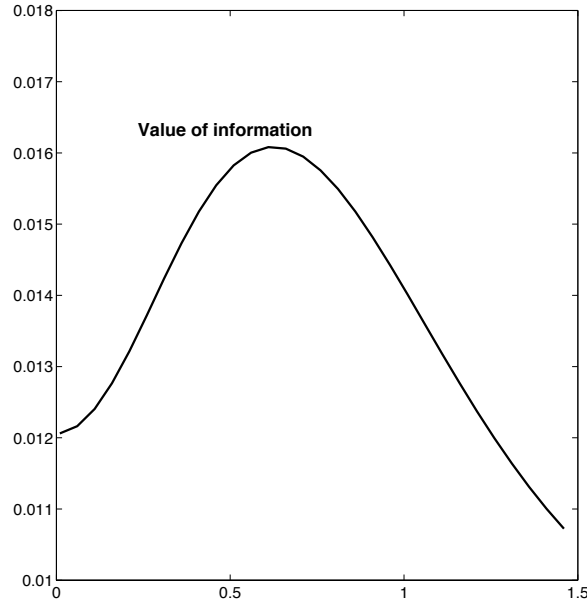


FIGURE 5. Value of information with a continuum of traders in each period.

Given the observation of  $Z_t$ , the public belief, which is identical to the price  $p_t$ , is given by<sup>14</sup>

$$p_t(Q_t; \lambda_t, p_{t-1}, \theta) = \frac{p_{t-1}G(Q_t; \lambda_t, \theta)}{p_{t-1}G(Q_t; \lambda_t, \theta) + 1 - p_{t-1}}, \quad (17)$$

with

$$G(Q_t; \lambda_t, \theta) = \exp\left(\frac{\lambda_t \Delta}{\omega^2} (\lambda_t (\theta - \bar{\theta}) + Q_t)\right). \quad (18)$$

An agent with signal  $s$  in the first period has a belief  $\tilde{\mu}(s, p_1)$  that is given by Bayes' rule:

$$\frac{\tilde{\mu}(s, p_1)}{1 - \tilde{\mu}(s, p_1)} = \frac{p_1}{1 - p_1} \exp\left(\frac{\Delta}{\sigma^2} (s - \bar{\theta})\right)$$

with  $\Delta = \theta_1 - \theta_0$  and  $\bar{\theta} = \frac{1}{2}(\theta_1 + \theta_0)$ .

Recalling that  $\lambda_2$  is fixed, his value of the financial asset at the end of period 1 is

$$A(s, p_1) = \tilde{\mu}(s, p_1)A_1(p_1) + (1 - \tilde{\mu}(s, p_1))A_0(p_1)$$

with

$$A_1(p_1) = \int p_2(Q_2; \lambda_2, p_1, \theta_1) f(Q_2) dQ_2,$$

$$A_0(p_1) = \int p_2(Q_2; \lambda_2, p_1, \theta_0) f(Q_2) dQ_2,$$

<sup>14</sup> $G(Q_t; \lambda_t, \theta) = \exp\left(-\frac{1}{2\omega^2} \left((Z_t - \lambda_t \theta_1)^2 - (Z_t - \lambda_t \theta_0)^2\right)\right) = \exp\left(\lambda_t \frac{\theta_1 - \theta_0}{\omega^2} \left(Z_t - \lambda_t \frac{\theta_1 + \theta_0}{2}\right)\right).$

where  $p_2(Q_2; \lambda_2, p_1, \theta)$  is given in (17).

The ex post value of information in period 1 is  $W(s, p_1) = |\text{sign}(A(s, p_1) - p_1)|$ .

Replacing  $\lambda_1$  by  $\lambda$ , the value ex ante of the information before period 1 is

$$\begin{aligned} V(\lambda; p_0) &= E[W(s, p_1) | \lambda, p_0] \\ &= p_0 V_1 + (1 - p_0) V_0, \end{aligned}$$

with

$$V_i = \int W(\theta_i + \epsilon, p_1(Q_1; \lambda, p_0, \theta_i)) \phi(\epsilon) f(Q_1) d\epsilon dQ_1,$$

where  $\phi$  and  $f$  are the densities of  $\epsilon$  and  $Q_1$ . The function  $V(\lambda; p_0)$  is computed numerically for the following values of the parameters:  $\omega = 1$ ,  $\sigma = 0.5$ ,  $\lambda_2 = 1$ ,  $p_0 = 0.92$ .

The graph of the function is represented in Figure 5. If the investment in private information is not too large, the value of information is increasing and there is strategic substitutability in the acquisition of information. When  $\lambda$  is large, the function  $V(\lambda)$  is decreasing as in the model of sequential trading in the text, and for the same reason. One can also verify that the range of  $\lambda$  for which  $V$  is increasing is larger when the price  $p_0$  increases towards 1. (The level of the value of information is obviously smaller.) For the parameters of Figure 5, there is some strategic complementarity if  $p_0 > 0.75$  or  $p_0 < 0.25$ .

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